

# GOLDEN ROOT GEOMETRY STRUCTURING THE POLYHEDRA AND OTHER FORMS VIA PLATO'S TRIANGLES

*Quadrature of Circle*

**Panagiotis Chr. STEFANIDES<sup>1</sup>**

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**ABSTRACT:**

*UNDER GOLDEN ROOT GEOMETRY STRUCTURING THE POLYHEDRA AND OTHER FORMS VIA PLATO'S TRIANGLES, WE REFER TO THE BASIC GEOMETRIC CONFIGURATIONS WHICH, AS THIS THEORY CONTEMPLATES, ARE NECESSARY FOR THE PROGRESSIVE MODE OF FORMATION OF THE FIVE POLYHEDRAL AND THE GEOMETRIES INVOLVED IN THEIR SECTIONS AND RELATED CIRCLES AND FURTHER TO LOGARITHMS, VIA LINES, AREAS AND VOLUMES. BASIS OF ALL THESE STRUCTURES IS A VERY SPECIAL SCALENE ORTHOGONAL TRIANGLE "PLATO'S MOST BEAUTIFUL" [F25], TOGETHER WITH HIS ORTHOGONAL ISOSCELES ONE. STRUCTURAL FORMS ARE IDENTIFIED BEARING IN COMMON THESE TRIANGULAR IDENTITIES. THE PARTICULAR ANGLE OF THE SCALENE ORTHOGONAL IS THAT WHOSE ARCTAN[Θ]=T AND  $T = \sqrt{(\sqrt{5} + 1)/2}$*

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**KEYWORDS:** THE MOST BEAUTIFUL TRIANGLE, ORTHOGONAL SCALENE TRIANGLE, ORTHOGONAL ISOSCELES TRIANGLE, SOMATOIDES TETRAHEDRAL STRUCTURE, GREAT PYRAMID MODEL, POLYHEDRA, CIRCLES QUADRATURE, SPIRALS, SPRIALOGARITHM.

## INTRODUCTION

By "Golden Root Geometry" we refer to two configurations of triangles. A Special one, the Quadrature Scalene Orthogonal Triangle [Author's interpretation of the Timaeic "Most Beautiful Triangle"] with sides  $[T^3]$ ,  $[T^2]$  and  $[T^1]$  in geometric ratio  $T$ , which is the square root of the golden ratio  $[\Phi]$ , and the Isosceles Orthogonal Triangle, with its equal sides  $[T]$ . The surface areas of these triangles are taken perpendicular to each other and in such, naturally, defining an X, Y, and Z system of coordinate axes. In so, the coordinates of the first are  $[0,0,0]$ ,  $[0,0,T^2]$ ,  $[T,0,0]$  in the X-Z plane, and those of the second are  $[0,0,0]$ ,  $[T,0,0]$ ,  $[0,T,0]$  in the X-Y plane. A line from  $[0,T,0]$  to  $[0,0,T^2]$ , creates the same Scalene Triangle in the Y, Z plane. ArcTan  $[T]$  is the Scalene angle  $[\Theta]$  of the Special Triangle with

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the property that the product of its small side by its hypotenuse is equal to the square of its bigger side:  $[T^1] \cdot [T^3] = [T^2]^2$  [Quadrature].

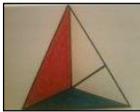
Using a pair of the Special Scalene Triangle, and a pair of a Similar Kind of Triangle [Constituent of the Special] with sides 1, T and  $T^2$  [Kepler/(Magirus) Triangle with sides 1,  $\sqrt{\Phi}$ , and  $\Phi$ ] a Tetrahedron [dicta Form 1- F4, Somatoides] is



F1 [Lines]



F2 [Areas- Triangles]



F3 [Volume-3D Space]



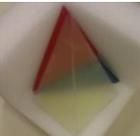
F4 [Form 1- red]



F5 [Form 2 - blue]



F6 [Form 3 - yellow]



F7 [F4+F5+F6]



F8



F9



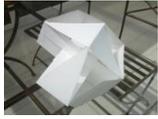
F10 [Back of F10]



F11 [ Back of F9]



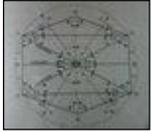
F12 [Icosahedron]



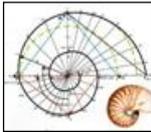
F13 [Dodecahedron]



F14 [Polyhedra]



F15 [Water Section]



F16 [Nautilus]



F17 [Spiral logarithms]



F18 [Volume 3D Space]



F19

obtained, by appropriately joining the edges of the four triangles [ F4], with coordinates:  $[0,0,0]$ ,  $[0,0,T^2]$ ,  $[T,0,0]$  and  $[0, 1/T, 1/T^2]$

By joining, a line, from point  $[0,T,0]$  to point  $[T,0,0]$ , a Second Tetrahedron [dicta Form 2 – F5] is obtained [ as a natural extension of Form 1], with co-ordinates:  $[0,0,0]$ ,  $[T,0,0]$ ,  $[0,T,0]$  and  $[0,1/T, 1/T^2]$ , having as base, on the X-Y plane, the Isosceles Orthogonal Triangle mentioned above, with coordinates  $[0,0,0]$ ,  $[T,0,0]$  and  $[0,T,0]$ .

Doubling this triangle, in the X-Y plane, a square is obtained of side  $[T]$ , with coordinates  $[T,T,0]$ ,  $[T,0,0]$ ,  $[0,0,0]$ , and  $[0,T,0]$ .

By connecting a line from point  $[T,T,0]$  to point  $[0,0,T^2]$  a third Tetrahedron [dicta Form 3 – F6] is obtained with coordinates:  $[T,T,0]$ ,  $[T,0,0]$ ,  $[0,T,0]$  and  $[0,0,T^2]$ ,

having also as base, the Isosceles Orthogonal Triangle with same dimensions [mirror image] as that of [Form 2 – F5]. The three Forms are wedged firmly together, leaving no empty space between them. Their volume ratios Form 3: Form 1: Form 2 equal to  $[1/6]*[T*T*T^2]$ :  $[1/6]*[1*T*T]$ :  $[1/6]*[T*T*(1/T^2)]$  is the golden ratio  $[T^2]$ , and the sum of volumes of Form 1 and Form 2 equals to  $[1/6]*[1*T*T] + [1/6]*[T*T*(1/T^2)]$  equals to  $[1/6]*[T^2+1]$  equal  $[1/6]*T^4$  [SINCE  $T^4-T^2-1=0$ ], the volume of Form 3. The volumes of the three Forms sum up to  $[(2/6)T^4]$  equal to  $(1/3)T^4$ .

Two of the four bases of Form 3, are symmetrical orthogonal triangles, with coordinates  $[T,0,0]$ ,  $[T,T,0]$ ,  $[0,0,T^2]$  and  $[T,T,0]$ ,  $[0,T,0]$ ,  $[0,0,T^2]$ , each of which has an angle  $[\varphi]$ , equal to  $\arctan[T^2]$ .

Two such triangles joined in a coplanar manner, and symmetrically along their bigger vertical sides, create one of the four triangular faces of a great pyramid model with coordinates  $[T, T, 0]$ ,  $[0, 0, T^2]$  and  $[T, (-T), 0]$ .

The Structure of the Three Forms bound together [dicta Form 4 –F7] with Volume  $[1/3]*T^4$  is one quarter of the volume of a great pyramid model [F8], which has a square base of side  $2T$ , height  $T^2$  and Volume  $[4/3]*T^4$ .

Splitting one of this model's triangular face into two orthogonal coplanar triangles to form a parallelogramme [with sides  $T^1$  and  $T^3$ ], we have constructed the basic skeleton of the Icosahedron [F12], since three such parallelogrammes, orthogonal to each other, determine its twenty equilateral triangle bases, by joining adjacent corners in groups of three, by lines.

Similarly, we proceed to the construction of the dodecahedron, the tetrahedron, the octahedron and the cube, together with their related forms such as squares, circles, triangles, circumscribed circles to the parallelogrammes of the polyhedra skeletons, circumscribed spheres and logarithmic spirals [F14 ], [ F15].[F16],[F17], [F20].

Reversing the whole process, the volumes decompose to the areas of the triangle surfaces structuring them which, in turn, resolve to four line traces harmonically codified in space [F19].

## II. PLATONIC TIMAEUS TRIANGLES

The work as described above follows, according to my interpretation of Plato's Timaeus description of "The Most Beautiful Triangle" and further, basing on this, the structure of his "world" of his Polyhedra. Lines of triangles represent elements [combinations of the 4 philosophical ones: Fire, Air, Earth and Water]. Solids created bear the same names, but include a further solid the "Fifth Consistency" according to Plato's word, for the Dodecahedron [ Aether].

### IIA. Sections 53-54 of Timaeus

According to Plato's Timaeus, ....The conditions prevailed before the world was created, while all elements [FIRE, AIR, EARTH and WATER] were "WITHOUT PROPORTION" [alogos] and "WITHOUT MEASURE" [ametros], and only "TRACES" of them existed, as all things, naturally exist in God's absence. God, under these conditions, transformed them via "IDEAS" and "NUMBERS", for them to become "MOST BEAUTIFUL" and "BEST" as possible, contrary to their previous state. ....

.... Πρώτον μεν δη πυρ και γη και ύδωρ και αήρ, ότι σώματά εστί..... τρίγωνα πάντα εκ δυοίν άρχεται τριγώνοιν.... προαιρετέον ούν άύ των απείρων το ΚΑΛΛΙΣΤΟΝ..... ΤΡΙΠΛΗΝ ΚΑΤΑ ΔΥΝΑΜΙΝ ΕΧΟΝ ΤΗΣ ΕΛΑΤΤΟΝΟΣ ΤΗΝ ΜΕΙΖΩ ΠΛΕΥΡΑΝ ΑΕΙ" .....

In sections 53-54, of Plato's "Timaeus", Plato speaks about the triangular shapes of the Four Elements [traces existed in disorder –matter- before their harmonization by God], of their kinds and their combinations:

These Bodies are the Fire (Tetrahedron) the Earth (Cube), the Water (Icosahedron), and the Air (Octahedron). These are bodies and have depth. The depth necessarily, contains the flat surface and the perpendicular to this surface is a side of a triangle and all the triangles are generated by two kinds of orthogonal triangles: the "Isosceles" Orthogonal and the "Scalene" Orthogonal. From the two kinds of triangles the "Isosceles" Orthogonal has one nature. (i.e. one rectangular angle and two acute angles of 45 degrees), whereas the "scalene" has infinite (i.e. it has one rectangular angle and two acute angles of variable values having, these two

acute angles, the sum of 90 degrees). From these infinite natures we choose one triangle "The Most Beautiful". Thus, from the many triangles, we accept that there is one of them "The Most Beautiful". Let us choose then, two triangles, which are the basis of constructing the Fire and the other Bodies : "Το μεν ισοσκελές, το δε τριπλήν κατά δύναμιν έχον της ελάττονος την μείζω πλευράν αεί."

### **IIB. Proposed New Interpretation:**

One of these two is the "Isosceles" Orthogonal Triangle, the other is the "Scalene" Orthogonal Triangle, its hypotenuse having a value equal to the "Cube" of the value of its horizontal smaller side and having its vertical bigger side the value of the "Square" of its smaller horizontal side. The value of the smaller horizontal side is equal to the square root of the Golden Number, the ratio of the sides is equal, again, to the Square Root of the Golden Number (geometrical ratio) and the Tangent of the angle between the hypotenuse and the smaller horizontal side is also equal to the Square Root of the Golden Number ( $\Theta = 51.49-38-15-9-17-19-54-37-26-24-0$  degrees). The product of the smaller horizontal side and that of the hypotenuse is equal to the "SQUARE" of the bigger vertical side, and the following equation holds:  $T^4 - T^2 - 1 = 0$ ,  $T = \text{SQRT}[(\text{SQRT}(5) + 1)/2]$ . The Kepler [Magirus] Triangle is a similar one but not the same. By "THE MOST BEAUTIFUL TRIANGLE", Plato correlates the four elements (UNIFIED THEORY) through the General Analogies of their sides (Fire, Air, Earth and Water), i.e. Fire/Air is equal to Air/Water is equal to Water/Earth, to T, where T is equal to the SQUARE ROOT of the GOLDEN NUMBER:  $T = \text{SQR}((\text{SQR}(5) + 1)/2)$  (ό τι περ πύρ προς αέρα, τούτο αέρα προς ύδωρ, και ό τι αήρ προς ύδωρ, ύδωρ προς γήν, ξυνέδησε.....ουρανόν, Plato's Timaeus section 32).

### **IIC. Section 37 - 39 of Timaeus**

According to Plato [Timaeus 37 -39]: ...He planned to make a movable image of Eternity, He made an eternal image, moving according to number, even that which we have named Time.... He contrived the production of days and nights and months and years ...And these are all portions of Time; as "Was" and "Shall be" are generated forms of Time...Time, then, came into existence along with Heaven, to the end that having been generated together they might also be dissolved together...this reasoning and design...with a view to the generation of Time, the sun and moon....and the other stars,...."planets", came into existence for the determining and preserving of the numbers of Time..... [Loeb].

Trying to understand the way possible for elements [according to Plato: traces - i.e. matter] to be joined together so that they compose matter, according to the geometry presented in this paper, we come to the following, possible, scenario:

In a static, but vibrating, field [Aether- Plato's reference to a Fifth Consistency, which God used up entirely to "colourfully paint" everything i.e. means by which matter is illuminated reflecting light from source (Fire) - electromagnetic medium], conductive [massive - "traces" of] elemental lines with alternating bipolar charges moving in it by the action of the field, should result into alternating currents running within them.

Two such lines could be contacted electrically at the ends of each line, via their + and - charges, and similarly three lines [in the harmonically correct lengths] could form triangles [orthogonal according to my theory], and in such forming a surface. Similarly by joining two pairs of such triangular forms [electromagnetically attracted by the currents running within them] could create materialistic volumes [tetrahedra]. Continuing, by these similar actions of electromagnetic forces, the joining of these materialistic volumes [tetrahedra] could result into further building blocks of matter.

According to my geometric theory [pure classical geometry, based on the Square Root of the Golden Section] such materialistic volumes [tetrahedra] build a Great Pyramid Model via which the structure of the world of the 5 Platonic [or Euclidean] solids.

We note that Plato states that everything that is born, it is born by necessity due to a cause, because it is impossible for it to be born without a cause.

With respect to world's creation, Plato states that, according to his thinking after having performed some assumptions, that three things exist before Heaven's coming into creation: Being, Place and Becoming.

### III. Circle's Quadrature [F19, F20, F21]

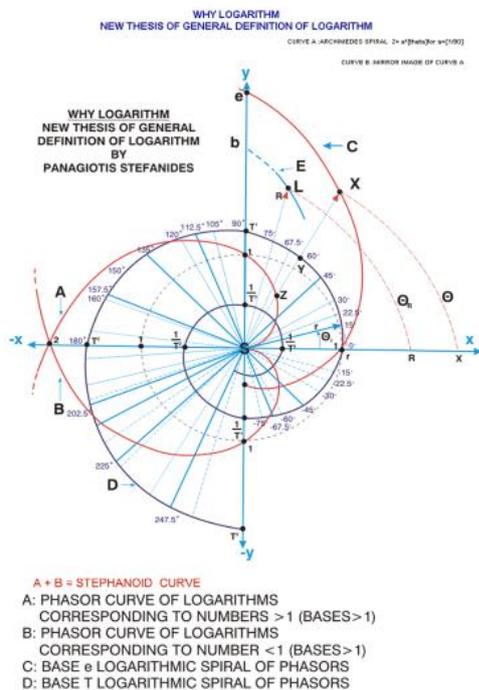
Structures built upon the form of the two Orthogonal Triangles, as described in this paper, have common relationships i.e. triangles, parallelograms, spirals, logarithms, squares, circles.

The main classical problem concerning circle relationships with the other geometric figures was that of its quadrature [conditions finding for its area to be equal to that of a square and its circumference equal to the perimeter of another square].

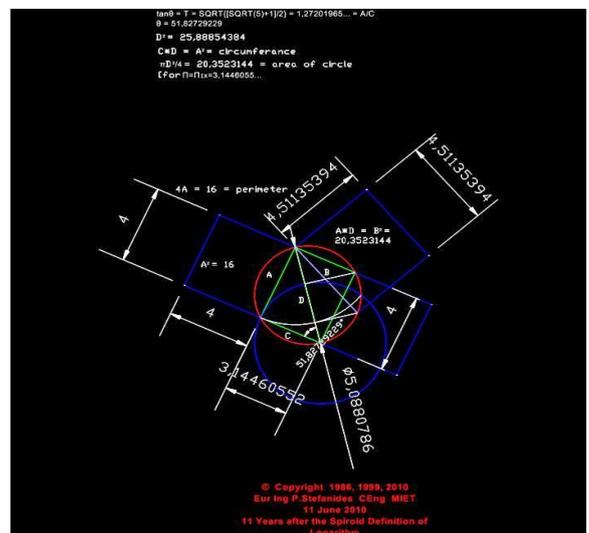
Plato's Scalene Orthogonal Triangle after lengthy elaboration, worked catalytically for forming a novel concept, for achieving the challenge of solving the "insoluble" problem of "quadrature of circle" [F21, F22, F23, ], proving that the value of Pi should be quantized :  $\pi = 4/[\text{SQRT}(\Phi)]$ , i.e. Pi equals 4 divided by the Square Root of the Golden Section [ = 3.14460551.....].

Further a [ PCST] Point on the Circle, the Square ,the Triangle - Maximum Symmetry Point [F24 ], was conceived for demonstration that the value of Pi should be quantized to the value of :  $4/[\text{SQRT}(\Phi)]$ , as achieved ,also, by ruler and compass [ F21, F22 ]. This serves as a gauge for estimating errors [shifts from PCST] of Pi values, used diachronically.

{ [F26] is the ruler and compass construction of angle  $\Theta$ , via  $\phi$  and  $\Phi$  }.

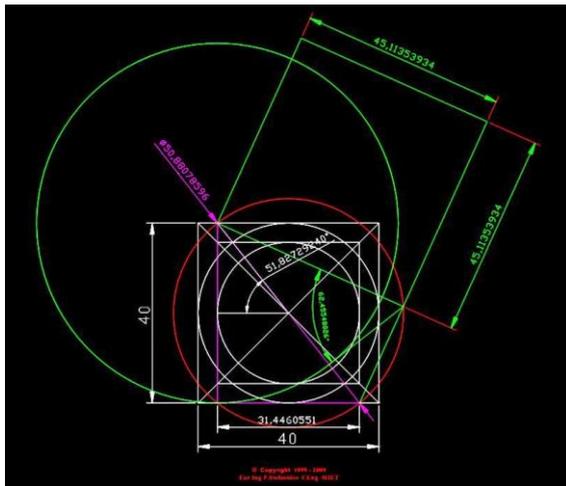


F20

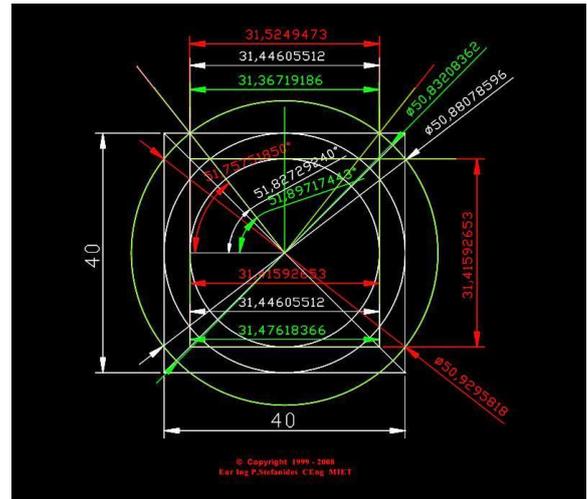


F21

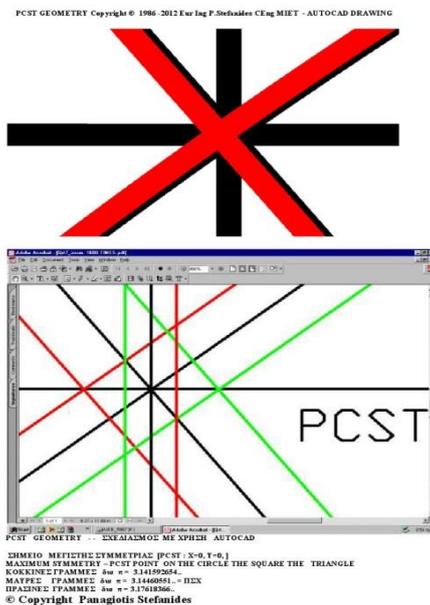
[F21, F22, F23, F24] AutoCAD Drawings  
 Vector Definition and Geometry Design by Panagiotis Stefanides, Computerized AutoCAD  
 by Dr. John Candyas



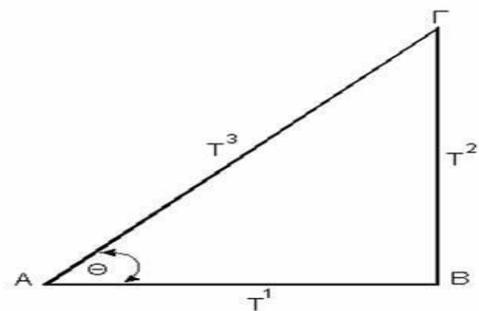
F22



F23

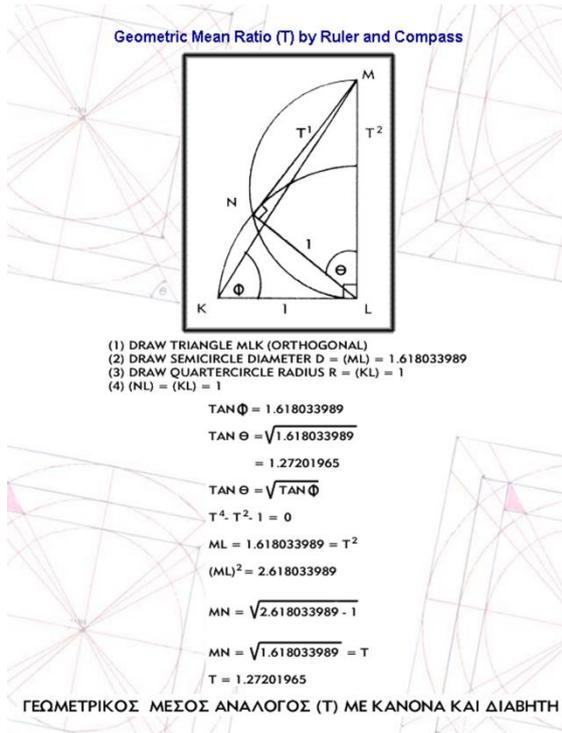


F24



$$\begin{aligned}
 T^4 - T^2 - 1 &= 0 \\
 T^6 - T^4 - T^2 &= 0 \\
 T^6 &= T^4 + T^2 \\
 (T^3)^2 &= (T^2)^2 + (T^1)^2 \\
 (A\Gamma)(AB) &= (\Gamma B)^2 \\
 \tan \Theta &= \frac{T^2}{T^1} = T \\
 \Theta &= \tan^{-1}(T) \\
 \frac{A\Gamma}{\Gamma B} &= \frac{\Gamma B}{BA} = T
 \end{aligned}$$

F25



F26

### III. CONCLUSIONS

Via the Golden Root Geometry we get relationships of Geometric structures, Logarithms and Spirals. It is concluded that by "THE MOST BEAUTIFUL TRIANGLE", Plato correlates the elements (Unified Theory) through the general analogies of their sides i.e. Fire/Air is equal to Air/Water is equal to Water/Earth, is equal to T the Golden Root. Finally, we realize Plato's statement that all triangles derive from two Orthogonal Triangles the Isosceles and the Scalene - "The Most Beautiful".

## REFERENCES

- [1]. Panagiotis Stefanides “The Most Beautiful Triangle” [Greek], Mathematical Society 2-4 Mart 1989, National Research Institution of Greece, Athens. Conference “History and Philosophy of the Classical Greek Mathematics”.
  - [2]. Paper Presentation and Proceedings Publication, “Golden Root Symmetries of Geometric Forms” The journal of the Symmetry: Culture and Science, Volume 17, Numbers 1-2, 2006, pp 97-111. Editor: Gyorgy Darvas. Conference: SYMMETRY FESTIVAL 2006, BUDAPEST HUNGARY.
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## ADDENDUM

Relationship of the Geometric Theory of the Proposed Plato’s “Most Beautiful Triangle” with the Specially Derived Spiral Logarithmic Curve Form the “Spiralalgorithm”.

### Logarithm Spiroid Definition

A model, Logarithmic Spiral, Prototype Definition Of Logarithms

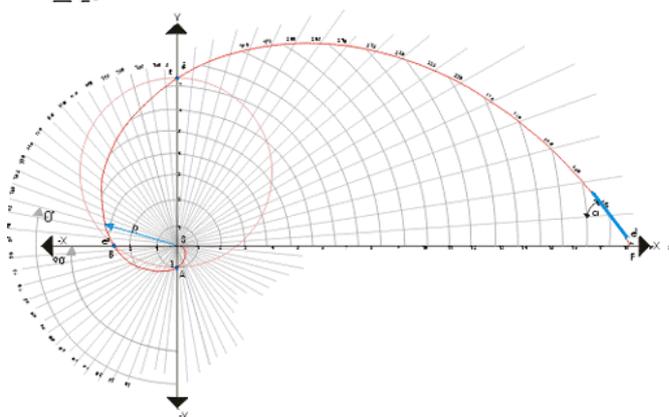
By definition  $\text{Log}_e(R) = 2 \left[ \frac{1}{2} \left( \frac{R}{R+1} \right)^2 + \frac{1}{2} \left( \frac{R}{R+1} \right)^4 + \frac{1}{2} \left( \frac{R}{R+1} \right)^6 + \dots \right]$

By this model theory:  $\text{Log}_e(R) = \left( \frac{R}{R+1} \right) = \left( \frac{R}{1+\text{TAN}a} \right) = \left( \frac{R}{1+\frac{d}{R}} \right) = \left( \frac{1}{\frac{1}{R} + \frac{d}{R^2}} \right) R$

And  $R = e^{\left( \frac{d}{R} \right)}$

$$\left( \frac{R}{R+1} \right) = \left( \frac{R}{1+\frac{d}{R}} \right)$$

$$\text{TAN}a = \frac{d}{R}$$

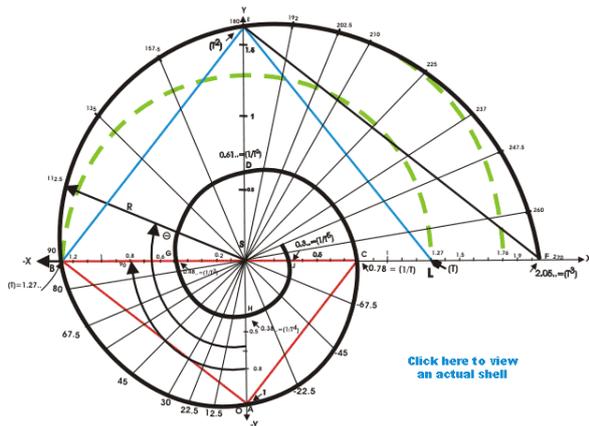


**APPLICATION EXAMPLES: MODEL FIXING VALUES:**

- 1) To Find Log of a number  
move a long circular arc,  
locate angle  $\Theta$  and divide by 90.
- 2) To find nth/root/power  
divide/multiply  $\Theta$   
by N and measure  
vector (R) along  
X-axis.
- 3) Multiplications / divisions  
by additions/subtractions.

$\left| \frac{SA}{SB} \right|$  Necessary pair, phased  $90^\circ$   
other points  $SE=e^2$  and  $SF=e^3$   
(e=Base of natural Logarithms  
It may be replaced for bases 10,2, etc.)

**NAUTILUS LOG [ T ] BASE SHELL CURVE**



NAUTILUS LOG BASE  $\sqrt{10}$  SHELL CURVE  
COPYRIGHT PANAGIOTIS STEFANIDES SEPT 2001  
 $T = \sqrt{10} = 1.27201965\dots$   
SET OF X - Y AXES  
CURVE CROSSES AXES AT:  
A =  $T \angle 0$  deg  
B =  $T \angle 90$  deg  
E =  $T \angle 180$  deg  
F =  $T \angle 270$  deg  
 $\log(R) = \frac{\Theta}{90}$   
 $\frac{\Theta}{90} = \frac{\Theta \text{ rad}}{(\frac{\pi}{2}) \text{ rad}} = \frac{(\frac{\Theta}{180})}{(\frac{\pi}{2})}$   
VECTOR SB = BASE (T)  
AT 90 DEG  
CLOCKWISE,  
FROM:  
VECTOR SA =  $T \angle 0$   
R, ANY VECTOR WITH  
ANGLE  $\Theta$ ,  
CLOCKWISE FROM SA  
POSITIVE, AND  
ANTICLOCKWISE  
NEGATIVE.

1. CURVE, APPROXIMATES, VERY CLOSELY,  
TO A NAUTILUS SHELL, FROM  
SYROS ISLAND (HERMOUPOULIS),2001.
2. NAUTILUS SHELL, FITS APPROX. WITHIN  
C D G H J, WITH DIMENSIONS (FACTOR 10):  
GC = 12.8 CM (THEORETICAL 12.7201965...)  
HD = 10.3 CM (THEORETICAL 10 CM)
3. THEORY FOLLOWS:  
LOGARITHM SPIROID DEFINITION  
<http://www.dotcreative.com/stefanides/logarithm.htm>  
<http://www.stefanides.gr>  
[panamars@otenet.gr](mailto:panamars@otenet.gr)
4. BASE(T), LOG EXAMPLE =  $\frac{210}{90} = 2.333\dots$ (THEOR: 2.34...)  
(R = 1.76):  $\log(1.76) = \frac{210}{90} = 2.333\dots$ (THEOR: 2.34...)
5. TRIANGLE ABC HAS SIDES :  $T^1, T^2, T^3$ , (PLATOS MOST  
BEAUTIFUL TRIANGLE, PROPOSED IN CONFERENCES BY  
P. STEFANIDES)  
TAN [(BCA), ANGLE J] = T (THEORETICAL)  
(BCA) ANGLE = 51 DEG, 49 FIRST, 38 SECOND.....  
(MODEL, GREAT PYRAMID (BEL) SECTION SLOPE)  
[Http://www.dotcreative.com/stefanides/piatostriangle.htm](http://www.dotcreative.com/stefanides/piatostriangle.htm)  
<http://www.dotcreative.com/stefanides/piato.htm>

<http://www.stefanides.gr/Html/logarithm.htm>  
<http://www.stefanides.gr/Html/Nautilus.htm>

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