CONSIDERATIONS ON THE USE OF MATHEMATICAL MODELING IN THE FORECAST OF ELECTRICITY CONSUMPTION

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ABSTRACT:
IN THIS PAPER WE AIM TO PROVIDE A SYNTHESIS OF THE MAIN METHODS USED TO ANALYZE THE EVOLUTION OF POWER CONSUMPTION IN ORDER TO HIGHLIGHT WAYS TO OPTIMIZE AND PREDICT IT. THE FIRST PART INCLUDES A GENERAL APPROACH TO THE CURRENT STATE OF EVOLUTION OF ENERGY SYSTEMS AND POLICIES BOTH IN A GLOBAL CONTEXT AND IN TERMS OF THE COMPLEXITIES OF THE ROMANIAN ENERGY SECTOR. ALSO, BY IDENTIFYING THE MAIN FACTORS OF INFLUENCE ON ELECTRICITY CONSUMPTION AS WELL AS THE DELIMITATION OF CONSUMER TYPES, WE WILL BE ABLE TO DEFINE GENERIC CONSUMPTION PROFILES AND ANALYZE THEIR ROLE IN CHOOSING THE TYPE AND METHOD OF FORECASTING ELECTRICITY CONSUMPTION. FINALLY, WE WILL PROVIDE A SYSTEMATIC OVERVIEW OF THE MATHEMATICAL METHODS USED IN FORECASTING ELECTRICITY CONSUMPTION, REVIEWING BOTH NUMERICAL METHODS AND TECHNIQUES, AS WELL AS MODERN APPROACHES BASED ON ARTIFICIAL INTELLIGENCE ALGORITHMS.

KEY WORDS: ELECTRICITY, MATHEMATICAL MODELING, FORECAST OF POWER CONSUMPTION

1. INTRODUCTION

Through dedicated mechanisms and instruments, the competitive market ensures the balance between the efficient usage of limited resources and the fulfillment of particular and social needs, that are unlimited and have a varying nature. The competitive relations adjust the producers' behavior by promoting discovery and technological progress as basic vectors of continuous and sustainable development. In this context, an efficient producer will aim to maximize his earnings, by incorporating into the restrictions system also constraints in order to ensure the optimization of consumption, thus improving the consumers' satisfaction level. Therefore, through a continuous adjustment and training process, the new challenges are turned into opportunities, thus ensuring the attaining of the whole economic system objectives.

The competitive market of electricity has certain characteristics generated both by the means of production, transport and/or distribution of electricity and also by its acquiring and consumption processes. The relationship between the producers and consumers of electricity, achieved through a predefined robust transport network, imposes the strong necessity of collaboration and integration between all the decision-making factors and action ones, in

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view of assuring and maintaining an internal system balance, but in the same time the necessity of a legislative framework, for regulating and coordinating at the macroeconomic level. It is common knowledge that electricity cannot be stored efficiently.

Therefore, the evaluation of the demand becomes an objective necessity for both the producers of electricity and for the distribution/supplying operators.

2. STATE OF ART REGARDING THE EVOLUTION OF THE PRODUCTIONAND CONSUMPTION OF ELECTRICITY

Among the general strategic objectives of the energy policies developed at the global, European and national level, one major concern consists in minimizing the difference, that tends to rise, between the production and consumption of electricity, by assuring an optimal mix of the energy resources, by correlating the economic development rate with the production capabilities and also by making the consumption more efficiently, in order to preserve and protect the environment.

Nowadays, an essential characteristic of the worldwide actual context, within which the energetic systems function, is represented by an increase in the energy consumption, especially the electricity, generated by many influence factors, like economic, technologic, demographic, climatic, regulatory, environmental ones etc.

In a more and more globalized context, the energetic policy of the European Union until the years 2020, relies on three fundamental objectives[3]:

- *sustainable development*–aims to improve the energy efficiency and reduce the negative impact of the energetic sector on the environment;
- *competitiveness* - aims developing competitive energy markets and assuring the nondiscriminatory access of all the participants to the energy market;
- *reliability in energy supply*- has as an objective the diversification of the energy sources, in view of reducing the vulnerabilities regarding potential future energetic crises.

The Romanian energetic policy must take into account the evolutions and changes that take place at the national level and the challenges at the global and European level in order to ensure the development of the national electric power system correlated to the strategic targets of the worldwide energy policies. According to the Romania's Energy Strategy for the period 2007-2020, the main target of the energy development sector consists in the "*covering the energetic needs both in the present and in the near and long future, at the lowest price, suitable to a modern market economy and to a civilized life level within quality conditions, security and maintaining the principles of sustainable development*" [15].

The particular aspects of the structure and dynamics of the national power system, and the strong links between this system and other activity branches, lead to the need of integrated approaching of phenomena and processes that develop within the whole chain of evolution from production, delivery/supplying and up to the energy consumption, in view of optimizing and obtaining efficiency.

In Romania, gross electricity consumption in the period 1990-2015 recorded a significant decrease, mainly due to the reduction in consumption in the industrial sector (Figure no. 1).

In the context of a broader market, the forecasting of the electricity consumption has an essential role in ensuring the efficient and safe operation of the energetic system, as an important link in ensuring the cybernetic system adjusts the processes along the whole chain of production – delivery/supplying of electricity.

[3] Energy 2020. A Strategy for Competitiveness, Sustainable and Secure Energy https://ec.europa.eu/energy/sites/ener/files/documents/2011_energy2020_en_0.pdf

In Romania, the process of the energy market liberalization brought a growth in the competition and therefore, an increase in the involvement of both the producers and electricity consumers, in order to identify the means of increasing the efficiency, in the context of the negative influence of the basic energy sources decreasing, of the instability of the electric power price, of the structural changes in the national economy, of the strong decreasing of the population number, but also taking into account the regulations regarding the environment protection.

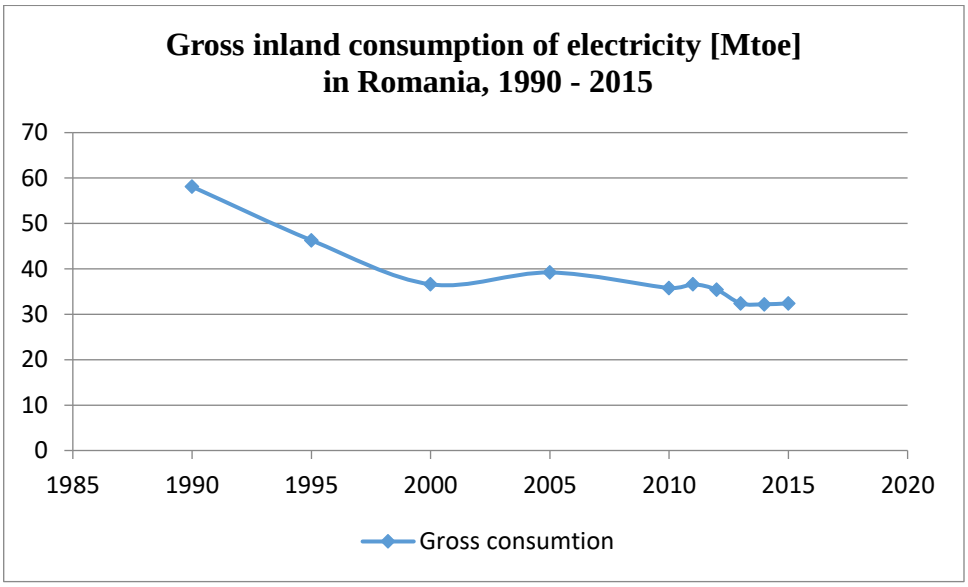

Figure no. 1

Source: Eurostat

3. INFLUENCE FACTORS OF THE ELECTRICITY CONSUMPTION

In the following, we will analyze the evolution of the electricity consumption taking into account the different mathematical modeling techniques, in view of finding the best methods and techniques for optimizing and forecasting that consumption.

The results obtained through the mathematical modeling of the electricity consumption reflect the real evolution of the process to an extent that essentially depends on the most accurate and complete identification of the influence factors that act upon it.

From this perspective, the main causal factors of electricity consumption are:

- *economic factors* - linked to the economy structure and its dynamics (GDP, the resident population, GDP per capita, number of consumers, price);
- *technological factors* - the technological changes imposed by the fast pace of development and innovation in the field of energy;
- *socio-demographic factors* - changes in the population dynamics and structure, in the urban/rural spatial distribution, in the living level evolution;
- *environmental factors*–the climate changes, increasing the share of renewable energy in total energy sources;
- *changes of the regulatory framework.*

The evolution of the Gross domestic product (GDP) has a decisive influence on the electricity demand, especially on the industrial energy consumption segment. Recent studies analyze even the opposite correlation, trying to highlight the contribution of electricity

consumption to economic growth, in the context of the structure's particularities and economic development at national level.

The changes brought about by the necessity of continuously adapting to the technological developments have a positive influence on all the participants in the electricity market, offering advantages both in terms of energy efficiency and intelligent monitoring of the electricity consumption. The existence of the two-way flow of information transmission between the producer and the consumer gives to the intelligent metering system *a feedback mechanism*, based on which realistic consumption forecasts can be made using dynamic load curve approximation methods[4].

Integrating and using intelligent metering systems into a power management system can help improve communication and control between consumers and suppliers, optimizing the electricity consumption, production and distribution.

The changes in the regulatory framework in view of aligning national laws with the Framework Strategies developed at a regional and global level facilitate the integration and cooperation between states, ensuring the security of the national electric power systems operation. In the same time, through the impact on the price, the legislative changes are a decisive factor influencing the evolution of the electricity consumption, also having direct consequences on the balancing process of the electricity market[5].

The climatic factors, along with the Energy Strategies' objectives generate changes both in the mix of the electricity generation sources and in the structure of its consumption in order to reduce the negative impact on the environment of the energy sector activities.

4. TYPES OF ELECTRICITY CONSUMERS AND GENERIC CONSUMER PROFILES

According to the current provisions of the electricity market regulatory framework, there are the following types of electricity consumers:

- *the end-user consumer*-consumes electricity on a contract basis and under regulated conditions may re-transmit electricity to subconsumers;
- *the household consumer* - uses electricity only for domestic purposes;
- *the public lighting system*–is under the coordination of the local administration;
- *the industrial electricity consumer*.

In the category of industrial electricity consumers, one can distinguish:

- *small consumers* - consumers having a contracted power of 100 kW or less on the consumption place;
- *large consumers* - consumers having a contracted power of over 100 kW on the consumption place

One of the major challenges of the liberalization of the electricity market process consists in ensuring the access to the electricity market for consumers who do not have an hourly consumption metering equipment (small consumers and household consumers). For these consumers, it is a question of determining the profile of the load curve, that is, the distribution of the total electricity consumption over a certain period of time over time intervals.

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[4] Căruțașu, George;Coculescu, Cristina;Stănică, Lavinia-Justina;Pîrjan, Alexandru; *An Analysis of the Main Characteristics and Implementation Requirements of the Advanced Metering Infrastructure Systems in Romania*, in Database Systems Journal, December 2016, ISSN: 2069-3230, pp. 354-363, ASE Edit Press, Bucharest, 2016

[5]Energy 2020. A Strategy for Competitiveness, Sustainable and Secure Energy https://ec.europa.eu/energy/sites/ener/files/documents/2011_energy2020_en_0.pdf; Romanian Energetic Strategy for the While 2007–2020. Updated to 2011-2020, http://www.minind.ro/energie/STRATEGIA_energetica_actualizata.pdf

By applying the methodology for determining the profile of load curves for electricity consumers, it is intended to group them into consumer classes having common characteristics and to establish the consumption profile of each class.

In the literature there are known two methods for determining the electricity consumption profile[6]:

- the statistical consumption profile method;
- the residual consumption profile method.

The statistical consumption profile method is based on complex procedures, analysis techniques, mathematical and statistical processing that ensure the obtaining of the most accurate results.

The residual consumption profile is computed as the difference between the total hourly consumption in a certain area and the total hourly consumption in that area of the consumers having smart meters installed that allow hourly consumption measurements. The method is simple to apply, but has the disadvantage that it does not offer the possibility of highlighting the peculiarities of the consumers in a particular residual group.

Although both ways of determining the consumption profile have advantages and disadvantages, in the specialized theory and practice is presented and applied with satisfactory results, especially the residual consumption profile method.

In Romania, starting with 2007, it is applied "The procedure for defining and utilizing the residual consumption profile", developed by the Romanian Energy Regulatory Authority (ANRE).

5. CLASSICAL METHODS VERSUS MODERN APPROACHES IN MATHEMATICAL MODELING OF ELECTRIC POWER CONSUMPTION

Current major and rapid transformations that address virtually all areas of life and socio-professional activities have led to the increasing complexity of processes and phenomena in society, which requires the use of modern methods of investigation combining the intuitive aspects with the scientific knowledge tools of reality, in view of obtaining decisions through deductions. The methods and techniques of forecasting have an important role in the process of scientific substantiation of managerial decisions, due to the multiple investigation and analysis possibilities that they offer[7].

In the scientific literature, there is no unitary and exhaustive classification of the mathematical modeling methods regarding the evolution of electricity consumption. Although scientific points of view are not always convergent, the following criteria for the classification of electricity consumption modeling and forecasting methods are distinguished[8]:

- Depending on the period to which the forecast refers:
  - short-term forecasting methods (from one hour to a week)
  - medium-term forecasting methods (from one week to one year)
  - long-term forecasting methods (more than one year)
- According to the mathematical method nature:
  - numerical methods (polynomial interpolation methods, finite differences, least squares adjustment methods)
  - optimization methods (linear, non-linear)

[6] Dumbravă, Virgil; Bazacliu, Gabriel; Nicoară, Bogdan; Popescu Christian; Apetrei, Dan;Using of Generic Profiles *of Consumption for Romanian Electric Power Market- The* Energetic, year 55, nr. 12/2007

[7] Coculescu, Cristina; Căruţaşu, George; Despa, Radu; *Information – Essential Element in Decision Process Modeling,* in Annals of the Oradea University, Fascicle of Management and Technological Engineering, CD-ROM Edition, Volume VII (XVII) 2008, pp. 2016_2021,Oradea, 2008

[8] Luștrea, Bucur; Energy Consumption Forecasting, AGIR Edit Press, 2001

- fuzzy methods
- algorithms of artificial intelligence

- According to the type of the method:
  - analytical methods (solution based on deduction)
  - simulation methods (having an experimental character)
- Depending on the nature of the independent variables:
  - direct methods
  - indirect methods

The regression method is one of the most used methods for the mathematical modeling of dependencies between two or more variables, especially the random ones. Starting from the experimental research of the analyzed phenomena variations and from the theoretical hypotheses formulated based on the principles and laws specific to the belonging scientific domain, the regression model describes quantitatively the dependencies between the specified variables. Furthermore, if the estimated regression model is valid, meaning that it reflects as accurately as possible the actual analyzed phenomena, it can be used to predict the evolution of the interest variables or to scientifically substantiate the managerial decisions.

In the literature, there are numerous studies in which the linear or non-linear regression method is presented and applied to describe the evolution of electricity consumption according to the main influence factors such as GDP, GDP per capita, number of consumers, price etc. Therefore, the theoretical electricity consumption regression model is given (1):

$$W_t = f(GDP_t, EP_t, NC_t, \dots.) \quad (1)$$

where: W_t is the electricity consumption, GDP_t is income or economic growth, EP_t is electricity price, NC_t is number of electricity customers etc.

The approaches aim, on one hand, to identify the most suitable mathematical function for describing the cause-effect correlations (the least squares method) and, on the other hand, to compare the regression method with other statistical and mathematical methods and techniques from the accuracy of the results point of view, but also from the possibilities to adapt to the level and volume of the processed data[9].

Some of the oldest forecasting methods are those based on time series analysis. The hypothesis in which these methods apply assumes that the studied phenomenon is comprised by the aggregation of the following components: the trend (reflects the long-term evolution of the studied phenomenon), the cyclical component, the seasonal component (periodicity due to the environment) and the residual component (including random fluctuations). The importance of these components depends on the period for which the forecast is made, as follows: short-term forecasts are very important for the residual component and less important for the trend and cyclical variations, while in the medium and long-term forecasts the trend component occupies the main place.

In power consumption evolution models, the aggregation of the four components is either additive (2) or multiplicative (3) with a variety of functions: linear, polynomial, exponential, Gompertz function, logistic function, etc.

[9] Bianco, Vincenzo; Manca, Oronzio; Nardini, Sergio; *Linear Regression Models to Forecast Electricity Consumption in Italy*, Journal Energy Sources, Part B: Economics, Planning and Policy, Volume 8, 2013-Issue 1; Mihai, Petruța; Popescu, O. Mihai;*Mathematical Models Used in Quality Management of the Electrical energy*, U.P.B. Sci. Bull., Series C, Vol. 70, No. 3, 2008

$$W_t = T_t + C_t + S_t + \varepsilon_t \quad (2)$$

$$W_t = T_t * C_t * S_t * \varepsilon_t \quad (3)$$

where: W_t is the electricity consumption, T_t is the trend, C_t *is* the cyclic component, S_t is the seasonal component and ε_t is the aleatory component.

The most frequently used methods for analyzing time series for the electricity consumption forecasting, which have led to satisfactory results from the practical point of view, are: ARMA, ARIMA, ARMAX, ARIMAX[10]. For the long-term electricity consumption forecast the methods should also take into account the changes induced by the technological development and innovation pace. The technological forecasting methods combine statistical and mathematical analysis tools with descriptive investigation techniques of phenomena and processes, among which the most popular are the trend curve method (exponential curves, linear, logistic, Gompertz curves) and the Delphi method (based on the experts' consultation, stimulating the scientific substantiation of forecasts).

Nowadays, in order to describe the dynamics of complex processes, as a result of the assembly of the component elements evolution, modern versions of the extrapolation methods are used. These methods are also known in the literature as "*extrapolation methods with the aid of the winding curve*". Operational research also provides a multitude of optimization algorithms for modeling and solving large scale problem shaving multiple objectives, as well as modern multicriterial analysis methods that provide forecasting in tight correlation with the overall macroeconomic developments, both in the energy field and in the systems with which they relate to.

The increasing complexity of processes and phenomena in the energy sector has required the development of modeling methodologies that include data processing methods based on *artificial intelligence* solutions: artificial neural networks, fuzzy logic, genetic algorithms. Due to the increased precision of the results, the possibility of finding complex data approximation functions, the generalization capacity and the modeling of the multivariable systems, there are several approaches to the electricity consumption forecasting using artificial neural networks and fuzzy techniques[11]. Also, one can remark the concern for the comparative estimation of electricity consumption for certain types of consumers, using both classical methods and models that incorporate artificial intelligence[12].

CONCLUSIONS

Forecasting the evolution of electricity consumption is of particular importance in the complex process of designing, developing and implementing modern management strategies, both in the field of production and in the one of electricity consumption, in accordance with the current constraints, requirements and challenges posed by the competitive environment integrating energy systems.

[10] Mohsen Mehrara; FatemehRafiei; *Energy Consumption and Economic Growth: Kalman Filter Approach*, International Journal of Academic Research in Economics and Management Sciences,May 2014, Vol. 3, No. 3 ISSN: 2226-3624; Pasapitch,Chujai; Nittaya, Kerdprasop; Kittisak; *Time Series Analysis of Household Electric Consumption with ARIMA and ARMA Models*, Proceeding of Soft International Multi-Conference of Engineers and Computer Scientists 2013VolI, IMECS2013,March 13-15, 2013,HongKong

[11] Ming, Meng; Wei, Shang; Dongxiao, Niu; *Monthly Electric Energy Consumption Forecasting Using Multi-window Moving Average and Hybrid Growth Models*, Journal of Applied Mathematics, Volume 2014 (2014), Article ID 243171

[12] Ozoh, P.; Abd-Rahman, S.; Labadin, J.; Apperley, M.; *A Comparative Analysis of Techniques for Forecasting Electricity Consumption*, International Journal of Computer Applications (0975 – 8887), Volume 88 – No.15, February 2014

The bibliographic research conducted in order to develop this paper regarding the methods and techniques used for optimizing and forecasting the electricity consumption, leads to the following assessments:

- the methods of modeling the electricity consumption evolution use a wide variety of statistical and mathematical tools and techniques (probability theory elements, numerical analysis, mathematical statistics, econometrics, artificial intelligence, etc.);
- both the classical methods and the dynamic methods of electricity consumption forecasting have advantages and limitations regarding the accuracy of the results, the flexibility, the applicability, the generalization capacity, the possibilities of computer implementation and integration in the forecasting software systems;
- in the case of the short-term electricity consumption forecasting, the research results have revealed particular influences on the energy consumption of the climatic factors (temperature, humidity, wind velocity etc.), but also the shape of the load curve for the special days;
- in the case of medium and long-term electricity consumption forecasting models it is used a range of macroeconomic factors such as: the economic growth, the population number, the consumer price index, the living standard, etc.;
- the accuracy of the forecasting results depends on the consumption profile, the accuracy of the data gathered from the consumption history, the technical characteristics of the software used for the implementation and data processing;
- the classical electricity consumption forecasting methods are relatively easy to apply and are recommended in the hypothesis of normalized measurement errors;
- the multiple (linear or non-linear) regression method is one of the most commonly used forecasting methods and provides good forecasting accuracy, especially for consumers who are faithful to a particular consumption profile.

The multitude of approaches and results mentioned in the literature reveals that using only classical numerical methods and techniques one cannot describe and analyze the varied and complex influences on electricity consumption, its peculiarities and interdependencies with various processes and phenomena in society. Recent studies highlight an increasing interest in using artificial intelligence methods and techniques (artificial neural networks, expert systems, fuzzy logic, genetic algorithms) in the mathematical modeling of the evolution of electricity consumption, since, due to the high degree of flexibility and adaptability, they lead to an increased accuracy of the forecasting results.

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13. Energy 2020. A Strategy for Competitiveness, Sustainable and Secure Energy https://ec.europa.eu/energy/sites/ener/files/documents/2011_energy2020_en_0.pdf
14. Romanian Energetic Strategy for the While 2007–2020. Updated to 2011-2020, http://www.minind.ro/energie/STRATEGIA_energetica_actualizata.pdf