Engineering

NEW APPROACH FOR FORWARD KINEMATICS MODELING OF INDUSTRIAL ROBOTS WITH CLOSED KINEMATIC CHAIN

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ABSTRACT:
MODELING THE FORWARD KINEMATICS FOR ROBOTS THAT INCLUDE CLOSE KINEMATIC CHAIN STRUCTURES IS DIFFICULT. WHEN DENAVIT-HARTENBER (DH) FORMALISM IS APPLIED, USUALLY IT COMES TO A SIMPLER KINEMATIC MODEL THAT DOES NOT ALWAYS CORRESPOND WITH THE REALITY. IF MORE THAN SIMPLY KNOWING THE END EFFECTOR POSITION IS DESIRED, SUCH AS KNOWING THE POSITION OF THE PASSIVE JOINTS IN THE CLOSED KINEMATIC CHAIN, THE DH CONVENTION IS NO LONGER SUFFICIENT. IN THIS PAPER, A NEW METHOD IS PRESENTED FOR FORWARD KINEMATIC MODELING FOR A ROBOT WITH A CLOSED CINEMATIC CHAIN (ABB IRB 460), AN EXTENDED MODELING METHOD THAT TAKES INTO ACCOUNT ALL ROBOT JOINTS (INCLUDING PASSIVE ONES) AND ALL THE ROBOT ELEMENTS (INCLUDING THOSE COMPOSING THE CLOSED CINEMATIC CHAIN). SUCH A MODEL PROVIDES BOTH POSITIONS OF THE END-EFFECTOR AND THE POSITIONS FOR ALL JOINTS (BOTH ACTIVE AND PASSIVE) AND CAN BE EASILY PARAMETERIZED TO TAKE INTO ACCOUNT SOME ERROR FACTORS (SUCH AS GEOMETRIC ERRORS AND THERMAL DEFORMATIONS) IN CASE IT IS DESIRED TO IMPLEMENT IT ON THE ROBOT CONTROLLER IN ORDER TO COMPENSATE THESE ERRORS.

KEY WORDS: PALLETIZING ROBOT, FORWARD KINEMATICS, CLOSED KINEMATIC CHAIN, NEW APPROACH.

INTRODUCTION

In the specific literature there are just few articles on the topic of forward kinematic modeling for robots with a closed kinematic chain structure (usually robots used in palletizing applications)[3]. Usually, the DH convention is used with applying of some simplifications, such as ignoring passive joints and structural elements that are part of the closed cinematic chain, leading to something similar to the modeling of the 4 DOF robots without a closed kinematic chain, fact that is not true in reality. In the idea that the robots are

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[3] Han, X.S.; Tian, Q.; *Kinematics Analysis of Palletizing Robot;* Advanced Materials Research, Vols. 915-916, ISSN: 1662-8985, pp. 477-481, Trans Tech Publications Ltd., 2014; Zhao, Y.G.; Xiao, Y.F.; Chen, T.; *Kinematics analysis for a 4-DOF palletizing robot manipulator*, Applied Mechanics and Materials, vol. 313-314, pp. 937–940, 2013; Zhu, S.X.; Lei, Q.S.; Li, J.Q.; *Analysis on influence coefficient of workspace about a configuration palletizing robot*, Advanced Materials Research, vol. 630, pp. 321–324, 2013

affected by errors, DH modeling (where the robot elements are considered to be rigid, and joint dimensions are considered null and where ambiguities exist due to location of the reference systems) no longer correspond to the real robot model and real behavior. As the result of researches carried out in[4] it was proved that by adding error parameters in the kinematic model, it can be established a mathematical model that better fits the robot than the nominal kinematic model. In the researches carried on the structural and functional optimization in order to increase the performance of an IR[5], 2 cases were considered for the elaboration of the analytical calculation model: 1) model of error unaffected IR (but including real constructive and functional parameters in addition to conventional DH parameters); 2) model of error affected IR (with modeling of the constructive and functional parameters in addition to the DH parameters and also the modeling of some error categories taken into account in the evaluation of the volumetric error). Such extended models have been studied, as in[6] where, based on DH modeling, a model with 29 parameters was used to calibrate an open-kinematic (ABB IRB140) robot to improve the positioning precision approximately 3 times. Error sources affecting the positioning accuracy of RI are presented in[7] and 5 factors responsible for robot errors have been identified: ambient (temperature), parametric (e.g. kinetic parameter variation due to friction or assembly errors, influence of dynamic parameters, friction and other nonlinearities, including hysteresis and backlashes), measurement (resolution and nonlinearities of joints position sensors), calculation errors (approximations and "steady state" errors) and application (installation errors). Further on, this paper presents a new approach (which already has been validated on 2 open-kinematic chain robots with symmetrical and non-symmetrical structures, ABB IRB120 and ABB IRB140,[8]) to develop a forward kinematic model for a robot with closed cinematic chain structures with the modeling of the closed kinematic chain elements and all passive joints.

TECHNICAL SPECIFICATIONS AND FUNCTIONAL PARAMETERS OF THE STUDIED ROBOT

The robot model is ABB IRB 460[9] and features a closed kinematic chain structure that ensures a horizontal position of the final flange through this mechanical synchronization system. This robot type is commonly used in palletizing operations. The robot has 5 degrees of freedom but only 4 are motorized (A - axis 1, B – axis 2, C – axis 3, D – axis 6). Main technical specifications and functional parameters may be found in ABB IRB 460 technical manual[10].

THE NEW APPROACH FOR FORWARD KINEMATICS

The first step in the realization of the forward kinematics is the elaboration of the wireframe kinematic scheme with the allocation of the joint reference systems and the identification of the necessary parameters for the homogenous coordinate transformation matrices. What can

[4] Georgia-Cezara, A.; *Cercetări privind optimizarea structural – funcţională a axelor comandate numeric ale RI, în vederea creşterii performanţelor acestora ,* PhD. Thesis, “Politehnica” University of Bucharest, 2014

[5] Georgia-Cezara, A.; *Cercetări privind optimizarea structural – funcţională a axelor comandate numeric ale RI, în vederea creşterii performanţelor acestora ,* PhD. Thesis, “Politehnica” University of Bucharest, 2014

[6] Nubiola, A.; Bonev I.A.; *Absolute calibration of an ABB IRB 1600 robot using a lasertracker,* Robot Comp IntegrManuf 2013;29(1):236–45

[7] Mehdi, C.; Jean-Yves K.; Alex B.; *Thermal aspects on robot machining accuracy,* Virtual Concept 2010 , Bordeaux, France

[8] Cristoiu, C.A.; Nicolescu, A.F.; *New approach for forward kinematic modeling of industrial robots;* Management and Innovative Technologies Conference – MIT2017; Sinaia, Romania, 2017

[9] ***ABB Product specification IRB460; Document ID: 3HAC039611-001;Copyright 2010-2017 ABB; www.abb.com/robotics

[10] ***ABB Product specification IRB460; Document ID: 3HAC039611-001;Copyright 2010-2017 ABB; www.abb.com/robotics

be seen on the kinematic scheme shown in fig. 1 (different from the DH convention) is the same orientation of the axis systems attached to each joint. To make it easier to understand for the moment just a simple model compatible with DH was developed (for now just few distances on Y axis between joints being taken into account, most of the joints being considered in the symmetry plane) following that in the near future the complete model to be developed with the real geometric parameters similar to the kinematic schemes presented in[11]. In the wireframe, represented in black color from L1 to L7 are specified the dimensions from the robot's datasheet and with the red color from L1p to L7p are specified dimensions measured on the virtual model of the robot and which are corresponding to the closed cinematic chain. The active joints are represented with a blue circle and the passive joints with red circles.

The mathematical modeling is still based on homogeneous coordinate transformations using successive transformations matrices from one joint to another. The general form of homogenous coordinate transformation matrix (4x4) composed from 4 separate sub-matrices, is the following:

$$T = \begin{pmatrix} R & p \\ \eta^T & \sigma \end{pmatrix} \qquad (1)$$

where: R = fundamental rotational matrix (3x3)

p = translation vector (3x1)

η = perspective vector (1x3) (in kinematics is the null vector)

σ = scale factor (usually = 1)

Steps for kinematic modeling: 1) first the translation matrices are applied on directions X, Y, Z from joint "i" to joint "i+1" (without any change in the orientation of the reference frame); 2) rotational matrix is applied for joint "i+1". Axis system of joint "i+1" will rotate around one of its own axis (depending which one is the rotation axis). Different from the modeling of the open-kinematic robots is only the multiplication with an additional rotation matrix (resetting in the normal position of some joints) as it is presented in the followings.

[11] Nicolescu, A.F.; Ilie, F.M.;Alexandru T.G.; *Forward and inverse kinematics study of industrial robots taking into account constructive and functional parameters modeling*; Proceedings in Manufacturing Systems, Volume 10, Issue 4, 2015, 157−16; ISSN 2067-9238

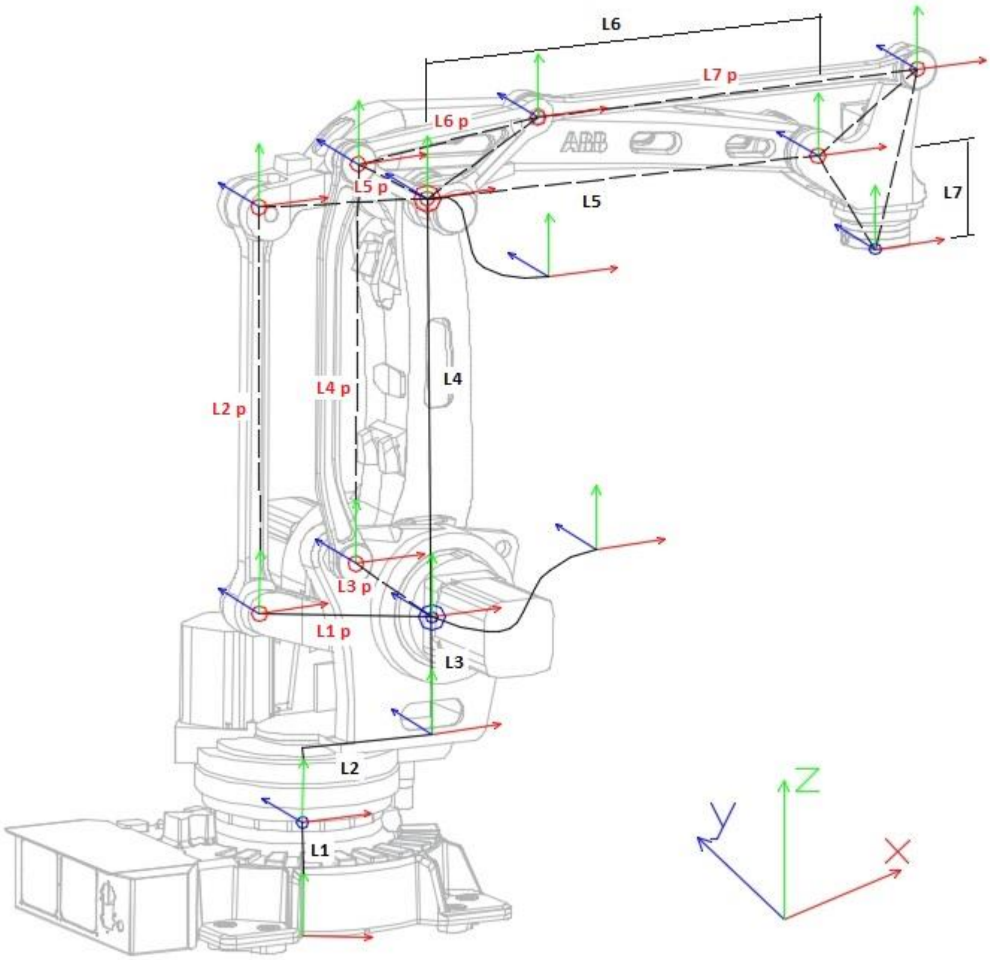

Fig. 1 Kinematic parameters and reference frames localization in the new approach

Robot dimensions used further as link parameters were taken from robot product manual[12].

Table 2: Parameters table for kinematic modeling

Rotation angles for active joints		Rotation angles for passive joints		Ref. axis translations for MAJOR joints				Ref. axis translations for PASSIVE joints			
					Pe X	Pe axa Y	Pe axa Z		Pe X	Pe axa Y	Pe axa Z
Θ_0	0	Θ_p1	0	Sis. Ref 0	0	0	0	Sis. Ref c.p 1	-400	0	0
Θ_1	0	Θ_p2	0	Sis. Ref 1	0	0	234.5	Sis. Ref c.p 2	0	0	945
Θ_2	0	Θ_p3	0	Sis. Ref 2	260	0	508	Sis. Ref c.p 3	-246.884	0	140.841
Θ_3	0	Θ_p4	0	Sis. Ref 3	260	0	508	Sis. Ref c.p 4	0	0	945
Θ_6	0	Θ_p5	0	Sis. Ref 4	0	0	945	Sis. Ref c.p 5	264.884	0	-140.841
		Θ_p6	0	Sis. Ref 5	1025	0	0	Sis. Ref c.p 6	229.813	0	192.836
		Θ_p7	0	Sis. Ref 6	220	0	-251.5	Sis. Ref c.p 7	1025	0	0

$\theta p1 = \theta 2-\theta 3$; $\theta p2 = \theta 3$; $\theta p3 = \theta 2$; $\theta p4 = \theta 3$; $\theta p5 = -1*\theta 2$; $\theta p6 = \theta 3$; $\theta p7 = 0$.

[12] ***ABB Product specification IRB460; Document ID: 3HAC039611-001;Copyright 2010-2017 ABB; www.abb.com/robotics

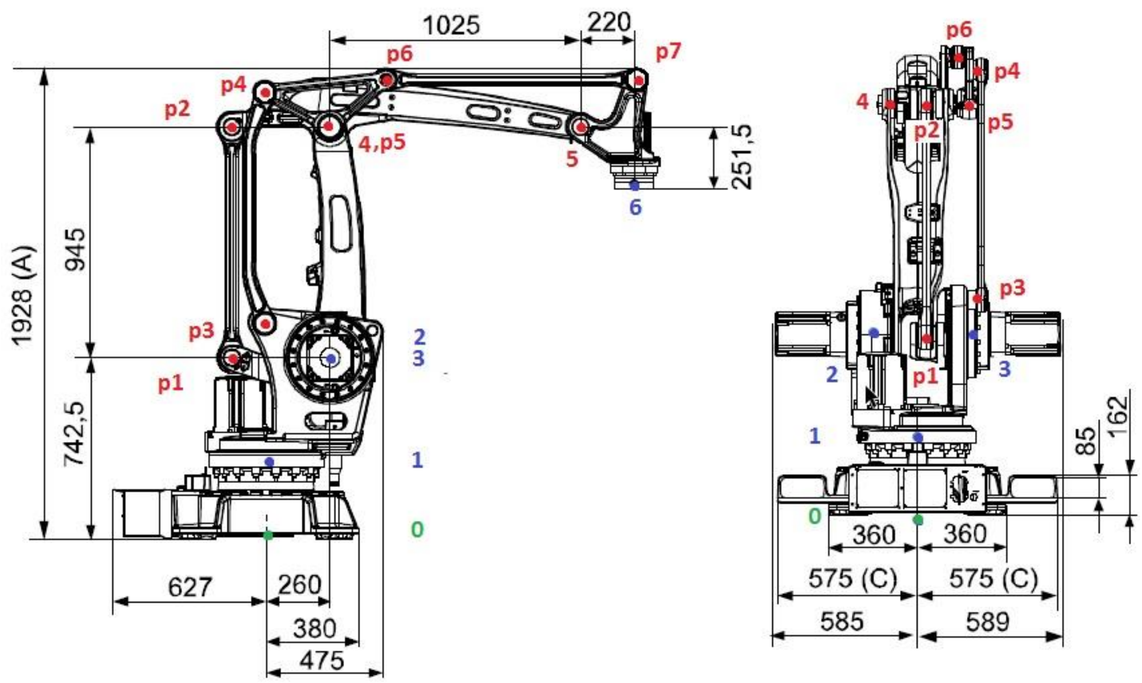

Fig. 2 Joint notations

Robot dimensions used further as link parameters were taken from robot product manual[13].

1) Transformation matrix from joint 0 to joint 1

$$\begin{pmatrix} cos\theta 1 & -sin\theta 1 & 0 & T.Sis.Ref1.X \\ sin\theta 1 & cos\theta 1 & 0 & T.Sis.Ref1.Y \\ 0 & 0 & 1 & T.Sis.Ref1.Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2) Transformation matrix from joint 1 to joint 2

$$\begin{pmatrix} cos\theta 2 & 0 & sin\theta 2 & T.Sis.Ref2.X \\ 0 & 1 & 0 & T.Sis.Ref2.Y \\ -sin\theta 2 & 0 & cos\theta 2 & T.Sis.Ref2.Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3) Transformation matrix from joint 1 to joint 3

$$\begin{pmatrix} cos\theta 3 & 0 & sin\theta 3 & T.Sis.Ref3.X \\ 0 & 1 & 0 & T.Sis.Ref3.Y \\ -sin\theta 3 & 0 & cos\theta 3 & T.Sis.Ref3.Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

4) Transformation matrix from joint 2 to joint

$$\begin{pmatrix} cos\theta 3-\theta 2 & 0 & sin\theta 3-\theta 2 & T.Sis.Ref4.X \\ 0 & 1 & 0 & T.Sis.Ref4.Y \\ -sin\theta 3-\theta 2 & 0 & cos\theta 3-\theta 2 & T.Sis.Ref4.Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

5) Transformation matrix from joint 3 to joint p1

$$\begin{pmatrix} 1 & 0 & 0 & T.Sis.Ref\ c.p\ 1.X \\ 0 & 1 & 0 & T.Sis.Ref\ c.p\ 1.Y \\ 0 & 0 & 1 & T.Sis.Ref\ c.p\ 1.Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

6) Transformation matrix to reset joint p1 to 0

$$\begin{pmatrix} cos\theta p1 & 0 & sin\theta p1 & 0 \\ 0 & 1 & 0 & 0 \\ -sin\theta p1 & 0 & cos\theta p1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

7) Transformation matrix from joint p1 to joint p2

11) Transformation matrix to reset joint 3 to 0

$$\begin{pmatrix} cos\theta 2-\theta 3 & 0 & sin\theta 2-\theta 3 & 0 \\ 0 & 1 & 0 & 0 \\ -sin\theta 2-\theta 3 & 0 & cos\theta 2-\theta 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

12) Transformation matrix from joint 3 to joint p5

$$\begin{pmatrix} 1 & 0 & 0 & T.(3\_p5).X \\ 0 & 1 & 0 & T.(3\_p5).Y \\ 0 & 0 & 1 & T.(3\_p5).Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Distances from joint 3 to joint p5 on X,Y,Z axis were measured on the virtual cad model.

13) Transformation matrix to reset joint p5 to 0

$$\begin{pmatrix} \cos(-\theta 2) & 0 & sin(-\theta 2) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(-\theta 2) & 0 & cos(-\theta 2) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

14) Transformation matrix from joint p5 to joint p6

$$\begin{pmatrix} 1 & 0 & 0 & T.(p5\_p6).X \\ 0 & 1 & 0 & T.(p5\_p6).Y \\ 0 & 0 & 1 & T.(p5\_p6).Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Distances from joint p5 to joint p6 on X, Y, Z axis were measured on the virtual cad model.

15) Transformation matrix to reset p6 to 0

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$$\begin{pmatrix} 1 & 0 & 0 & T.Sis.Ref\ c.p\ 2.X \\ 0 & 1 & 0 & T.Sis.Ref\ c.p\ 2.Y \\ 0 & 0 & 1 & T.Sis.Ref\ c.p\ 1.Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

8) Transformation matrix from joint 4 to joint 5

$$\begin{pmatrix} 1 & 0 & 0 & T.Sis.Ref5.X \\ 0 & 1 & 0 & T.Sis.Ref5.Y \\ 0 & 0 & 1 & T.Sis.Ref5.Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

9) Transformation matrix from joint 1 to joint p3

$$\begin{pmatrix} cos\theta p3 & 0 & sin\theta p3 & T.(1\_p3).X \\ 0 & 1 & 0 & T.(1\_p3).Y \\ -sin\theta p3 & 0 & cos\theta p3 & T.(1\_p3).Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Distances from joint 1 to joint p3 on X,Y,Z axis were measured on the virtual cad model.

10) Transformation matrix from joint p3 to joint p4

$$\begin{pmatrix} 1 & 0 & 0 & T.(p3\_p4).X \\ 0 & 1 & 0 & T.(p3\_p4.Y \\ 0 & 0 & 1 & T.(p3\_p4).Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Distances from joint p3 to joint p4 on X,Y,Z axis were measured on the virtual cad model.

$$\begin{pmatrix} os\theta 3 & 0 & sin\theta 3 & 0 \\ 0 & 1 & 0 & 0 \\ -sin\theta 3 & 0 & cos\theta 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

16) Transformation matrix from joint p6 to joint p7

$$\begin{pmatrix} 1 & 0 & 0 & T.(p6\_p7).X \\ 0 & 1 & 0 & T.(p6\_p7).Y \\ 0 & 0 & 1 & T.(p6\_p7).Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Distances from joint p6 to joint p7 on X, Y, Z axis were measured on the virtual cad model.

17) Transformation matrix to reset 5 to 0

$$\begin{pmatrix} cos\theta 2 & 0 & sin\theta 2 & 0 \\ 0 & 1 & 0 & 0 \\ -sin\theta 2 & 0 & cos\theta 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

18) Transformation matrix from joint 5 to joint6

$$\begin{pmatrix} cos\theta 6 & -sin\theta 6 & 0 & T.Sis.Ref6.X \\ sin\theta 6 & cos\theta 6 & 0 & T.Sis.Ref6.Y \\ 0 & 0 & 1 & T.Sis.Ref6.Z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

By multiplication of the frame transformation matrices for the right joints the solution of forward kinematics can be obtained in order to compute position and orientation for each joint or for the end-effector position. Position and orientation of each joint can be obtained from the previous equations as follows:

Joint 1 = T_{01}
Joint 2 = $T_{01}*T_{12}$
Joint 3 = $T_{01}*T_{13}$
Joint 4 = $T_{01}*T_{12}*T_{24}$
Joint 5 = $T_{01}*T_{12}*T_{24}*T_{45}$
Joint 6 = $T_{01}*T_{12}*T_{24}*T_{45}*T_{5\ reset}*T_{56}$

int p1 = $T_{01}*T_{13}*T_{3p1}*\ T_{p1\ reset}$
int p2 = $T_{01}*T_{13}*T_{3\ p1}*T_{p1\ reset}*T_{p1\ p2}$
int p3 = $T_{01}*T_{1\text{-}p3}*T_{p3\ reset}$
int p4 = $T_{01}*T_{1\text{-}p3}\ *T_{p3\text{-}p4}$
int p5 = $T_{01}*\ T_{13}*\ T_{3\ p5}*\ T_{p5\ reset}$
int p6 = $T_{01}*T_{13}\ *T_{3\ p5}*\ T_{p5\ reset}*T_{p5\ p6}*T_{p6\ reset}$
int p7=$T_{01}*T_{13}*T_{3p5}*T_{p5reset}*T_{p5p6}*T_{p6reset}*T_{p6\ p7}$

Robot's end flange characteristic point is obtained by multiplying coordinate transformation matrices from base to axis 6 as follows:

$$T_{base\text{-}end\ flange}=T_{01}*T_{12}*T_{23}*T_{34}*T_{45}*T_{56}=\begin{pmatrix} R11 & R12 & R13 & Px \\ R21 & R22 & R23 & Py \\ R31 & R32 & R33 & Pz \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (19)$$

VALIDATION OF NEW MODELING FORMALISM WITH 3D CAD MODEL

The virtual model of the ABB IRB 460 robot was downloaded from the manufacturer's CAD database. The files were imported in CATIA V5 software and then assembled in Assembly workbench. Joint definition and constrained were created in DMU Kinematics workbench. Here are also set the commands for joints movement and „Measure" tags are created for monitoring each joint position.

In order to compute each joint position, the mathematical model was implemented in an excel files with 35 matrices (18 transformation matrices + 17 matrices multiplications). Only the first and the last ones are presented in the following fig. 4 and 5.

On the virtual model, multiple joint commands were set up. For each different pose of the robot configured in the virtual model, same joint values were set as parameter in the excel file. For each pose, same results were obtained both in the virtual CAD model (fig. 6) and in the mathematical model of the new forward kinematic model implemented in excel (fig. 7). In fig. 6 and fig. 7 there are presented the initial results for the "Home" pose as well for a random configuration ($\theta1=150^0$, $\theta2=35^0$, $\theta3=40^0$, $\theta6=30^0$).

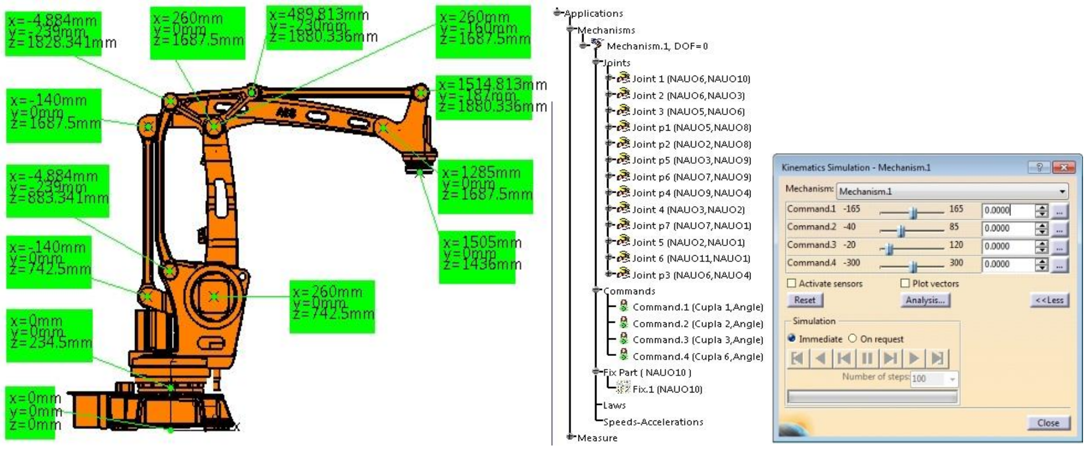

Fig. 3 Robot virtual model, measure tags, DMU Kinematics tree specification / command window

Datasheet dimensions

L1	234.5
L2	508
L3	260
L4	945
L5	1025
L6	220
L7	-251.5

Joint rotation angles

Θ_0	0	Θ_p1	0
Θ_1	0	Θ_p2	0
Θ_2	0	Θ_p3	0
Θ_3	0	Θ_p4	0
Θ_6	0	Θ_p5	0
		Θ_p6	0
		Θ_p7	0

Parameters for reference systems

Translations for MAJOR joints axis systems			
	Pe X	Pe axa Y	Pe axa Z
Sis. Ref 0	0	0	0
Sis. Ref 1	0	0	234.5
Sis. Ref 2	260	0	508
Sis. Ref 3	260	0	508
Sis. Ref 4	0	0	945
Sis. Ref 5	1025	0	0
Sis. Ref 6	220	0	-251.5

Translations for passive joints axis systems			
	Pe X	Pe axa Y	Pe axa Z
Sis. Ref c.p 1	-400	0	0
Sis. Ref c.p 2	0	0	945
Sis. Ref c.p 3	-246.884	0	140.841
Sis. Ref c.p 4	0	0	945
Sis. Ref c.p 5	264.884	0	-140.84
Sis. Ref c.p 6	229.813	0	192.836
Sis. Ref c.p 7	1025	0	0

Transf. matrix from 0_1

1	0	0	0
0	1	0	0
0	0	1	234.5
0	0	0	1

Transf. matrix from 1_3

1	0	0	260
0	1	0	0
0	0	1	508
0	0	0	1

Transf. matrix from 1_2

1	0	0	260
0	1	0	0
0	0	1	508
0	0	0	1

Transf. matrix from 2_4

1	0	0	0
0	1	0	0
0	0	1	945
0	0	0	1

T01*T12

1	0	0	260
0	1	0	0
0	0	1	742.5
0	0	0	1

T01*T12*T24

1	0	0	260
0	1	0	0
0	0	1	1687.5
0	0	0	1

T01*T13

1	0	0	260
0	1	0	0
0	0	1	742.5
0	0	0	1

T01*T13*T3_p1

1	0	0	-140
0	1	0	0
0	0	1	742.5
0	0	0	1

Joints coordinates							
Active	X	Y	Z	Passive	X	Y	Z
C1	0	0	234.5	p1	-140	0	742.5
C2	260	0	742.5	p2	-140	0	1687.5
C3	260	0	742.5	p3	-4.884	-239	883.34
C6	1505	0	1436	p4	-4.884	-239	1828.3
				4	260	0	1687.5
				p5	260	-160	1687.5
				p6	489.81	-230	1880.3
				p7	1514.8	-187	1880.3
				5	1285	0	1687.5

Fig. 4 Parameters table, first transformation and multiplication matrices and results table

Set p5 to 0 for transf. from p5 to p6

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Set p6 to 0 for transf. from p6 to p7

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Set 5 to 0 for. Transf from 5 to 6

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Transf. matrix from p5 to p6

1	0	0	229.813
0	1	0	-70
0	0	1	192.836
0	0	0	1

Transf. matrix from p6 to p7

1	0	0	1025
0	1	0	43
0	0	1	0
0	0	0	1

Transf. matrix from 5 to 6

1	0	0	220
0	1	0	0
0	0	1	-251.5
0	0	0	1

T01*T13*T3_reset*T3_p5*Tp5_reset*Tp5_p6

1	0	0	489.813
0	1	0	-230
0	0	1	1880.336
0	0	0	1

T01*...*Tp6_p7

1	0	0	1514.813
0	1	0	-187
0	0	1	1880.336
0	0	0	1

T01*T12*T24*T45*T5_reset*T56

1	0	0	1505
0	1	0	0
0	0	1	1436
0	0	0	1

T01*T13*T3_reset*T3_p5*Tp5_reset*Tp5_p6*Tp6_reset

1	0	0	489.813
0	1	0	-230
0	0	1	1880.336
0	0	0	1

T01*T12*T24*T45*T5_reset

1	0	0	1285
0	1	0	0
0	0	1	1687.5
0	0	0	1

Fig. 5 Last joint transformation and multiplication matrices and final flange position

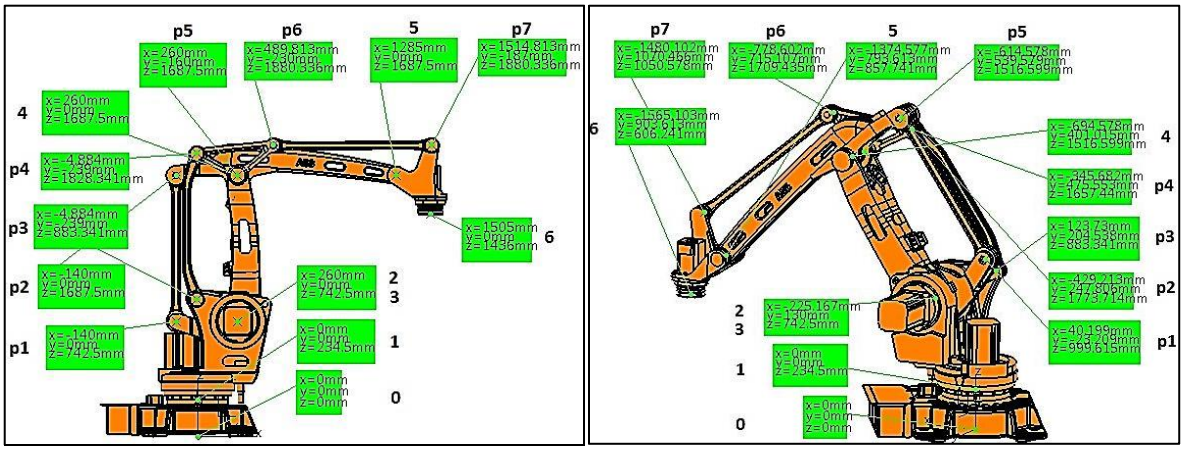

Fig. 6 Virtual model results for “Home” pose and random pose

Coordonate cuple							
Active Joints	X	Y	Z	Passive Joints	X	Y	Z
C1	0	0	234.5	p1	-140.000	0.000	742.500
C2	260	0	742.5	p2	-140.000	0.000	1687.500
C3	260	0	742.5	p3	-4.884	-239.000	883.341
C6	1505	0	1436	p4	-4.884	-239.000	1828.341
				4	260.000	0.000	1687.500
				p5	260.000	-160.000	1687.500
				p6	489.813	-230.000	1880.336
				p7	1514.813	-187.000	1880.336
				5	1285.000	0.000	1687.500

Coordonate cuple							
Active Joints	X	Y	Z	Passive Joints	X	Y	Z
C1	0	0	234.5	p1	40.199	-23.209	999.615
C2	-225.166605	130	742.5	p2	-429.213	247.806	1773.714
C3	-225.166605	130	742.5	p3	123.730	204.538	883.341
C6	-1565.103009	903.6126433	606.2413819	p4	-345.682	475.553	1657.440
				4	-694.578	401.015	1516.599
				p5	-614.578	539.579	1516.599
				p6	-778.602	715.107	1709.435
				p7	-1480.101	1070.466	1050.577
				5	-1374.577	793.613	857.741

T01*T12*T24*T45*T5_reset*T56

0.866	-0.5	0	1505
0.5	0.866	0	0
0	0	1	1436
0	0	0	1

T01*T12*T24*T45*T5_reset*T56

-1	-6E-17	6E-17	-1565.10301
-6E-17	-1	-3E-17	903.6126433
0	0	1	606.2413819
0	0	0	1

Fig. 7 Mathematical model results for “Home” pose and random pose

CONCLUSIONS

In this paper a new approach was presented regarding the elaboration of the mathematical model for the forward kinematics of robots with closed cinematic structures (as is usually the case with robots dedicated to palletizing operations). This approach also involves modeling the passive joints and taking into account the elements of the closed kinematic chain. For the first time, a mathematical model is presented in which not only the position of the active joints but also the position of the passive joints is monitored. This is of great importance if there is a need to develop a model that takes into account real functional constructive parameters. For example, the thermal deformations that may occur in the elements of the closed kinematic chain structure can seriously affect the positioning accuracy of the robot as well as the orientation of the axis 6 which for this type of robots must mandatory be in a vertical position to ensure the end flange parallelism with the ground. Applying the approach presented in this paper facilitates the development of a complete mathematical model, where error parameters can easily be introduced as simple displacements in the parameter table (chapter 3, Table 2). This method is also more easily to be applied by using reference frames with same orientation on the robot's entire structure. This fact is also avoiding DH ambiguities about positioning of reference frames and provides a clear general method to be used by everyone. The mathematical results were computed within an excel file and compared with the robot's virtual model. The results were identical proving the new method is correct.

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