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Authors:	Muhammed DEMİR

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NEW FIBONACCI SERIES THE OTHER SIDE OF THE COIN

Muhammed DEMİR[1]

ABSTRACT:
IT IS KNOWN THAT THE FIBONACCI SEQUENCE IS USED IN DOZENS OF FIELDS SUCH AS FINANCE, NATURE, ART, CRYPTOGRAPHY, MUSIC AND ECONOMY. THEY ARE FINDING THE (N)TH ELEMENT IN THE FIBONACCI SEQUENCE REQUIRES CALCULATING THE (N-1) AND (N-2) ELEMENTS. THE FIRST AND SECOND ELEMENTS ARE CALCULATED RECURSIVELY. THE FIBONACCI SEQUENCE IN THIS STUDY IS ASCERTAINED USING THE PASCAL TRIANGLE. A FORMULA FOR DIRECTLY DETERMINING THE NECESSARY FIBONACCI ELEMENT HAS BEEN PROPOSED. WHEN ALL OF THE ELEMENTS IN THE DIAGONAL PLANE ARE GATHERED IN PASCAL TRIANGLES FROM LEFT TO RIGHT, IT IS KNOWN THAT THE ELEMENTS OF THE SEQUENCE KNOWN AS FIBONACCI CAN BE CALCULATED SEQUENTIALLY. THE HIDDEN PATTERN WITHIN THIS TRIANGLE IS CONVERTED INTO A NEW FORMULA BY UTILIZING MATHEMATICAL FUNCTIONS, SIGMA SYMBOLS, AND COMBINATION MODELING. RECURSION AND DYNAMIC PROGRAMMING COMPUTATIONS YIELD THE MINIMAL TIME AND SPACE COMPLEXITY TO FIND THE FIBONACCI SERIES.

KEYWORDS: FIBONACCI SEQUENCE, GOLDEN RATIO, RECURSION, FORMULA, HISTORY OF MATH.

INTRODUCTION

Designed in the 13th century by the Italian philosopher Leonardo Fibonacci and included in his work "Liber Abaci", the Fibonacci sequence is used in numerous fields such as calculating the golden ratio [1]. This sequence of numbers emerged during the calculation of the problem related to the reproduction of rabbits. It is also stated that the Fibonacci sequence was basically discovered by Indian mathematical thinkers in the 6th century [2]. It is possible to define the Fibonacci number sequence mathematically as follows:

$$fib(n) = \begin{cases} 0 & , \quad n = 0 \\ 1 & , \quad n = 1 \\ fib(n-1) + fib(n-2) & , \quad n > 1 \end{cases}$$

In the Fibonacci sequence, to find the value of any n. element, the sum of the two elements preceding that element is taken. This rule can be used to calculate all elements in the

[1] Mathematics Teacher, Ministry of National Education, Turkey, muhammeddemir1618@gmail.com, ORCID: 0009-0009-7671-920.

sequence, provided that the first and second element have a value of 1. Also, when the n. element is denoted as Fn, it can be shown mathematically recursively as $F_n = F_{n-1} + F_{n-2}$, provided that $F_0=0$ and $F_1=1$. The first few elements of this sequence are as follows: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,... To reach n elements in this array, approximately n backward operations are required. The method of this study allows to easily find the proposed formula without the need for n operations.

Fibonacci number sequence is used in many fields such as computer science, economics [4] and biology [3]. For example, the Fibonacci Search Method is an optimized and one-dimensional search method [5]. Also, Fibonacci Heap serves as a data structure model [6]. In graph theories, the Fibonacci cube is used as a network topology that connects distributed and parallel systems [7].

In computing and mathematics, the Fibonacci coding technique can be used to convert positive integers into binary number systems [8]. Each positive integer can be written as the sum of one or more different Fibonacci numbers. In addition, no Fibonacci numbers in this method are used in order. The Zeckendorf Theorem is a short explanation based on the Fibonacci Coding technique [9].

At the same time, the runtime computation in the Euclidean Algorithm found the greatest common divisor of two integers (EBOB). The last two elements of the Fibonacci number sequence represent the worst case [10].

The method, also known as Pseudorandom Number Generator (PRNG), utilizes the Fibonacci number sequence. This method involves deriving a sequence of numbers that makes it difficult to form relationships or patterns between elements [11]. Planning Poker or Scrum Poker is a different game that uses the Fibonacci number sequence. Planning Poker is used as a goal setting game in software development [12].

Another method that utilizes the Fibonacci number sequence is multiphase combinatorial search. This method recursively divides the numbers in a non-sequential sequence into binary groups at each iteration. The length of the groups is determined by consecutive Fibonacci elements. This brings the ratios between groups closer to the golden ratio [13].

Programming languages can be used to find the elements of the Fibonacci sequence. The elements of this sequence can be found by dynamic programming and recursion methods. The C/C++ snippet for recursively finding the Fibonacci sequence is as follows:

```
int fibonacci(int n)
{
If (n == 0)
    return 0;
else if(n == 1)
    return 1;
else
return fibonacci(n - 1) + fibonacci(n - 2);
}
```

This sequence of numbers can also be found by the bottom-up method used in the dynamic programming language. In general, this method turns into a linear method and can be found by the following method (Figure 1):

```
int fibonacci(int n)
{
int f[n+1];
f[1] = f[2] = 1;
for (int i = 3; i <= n; i++)
```

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```
f[i] = f[i-1] + f[i-2];
return f[n];
}
```

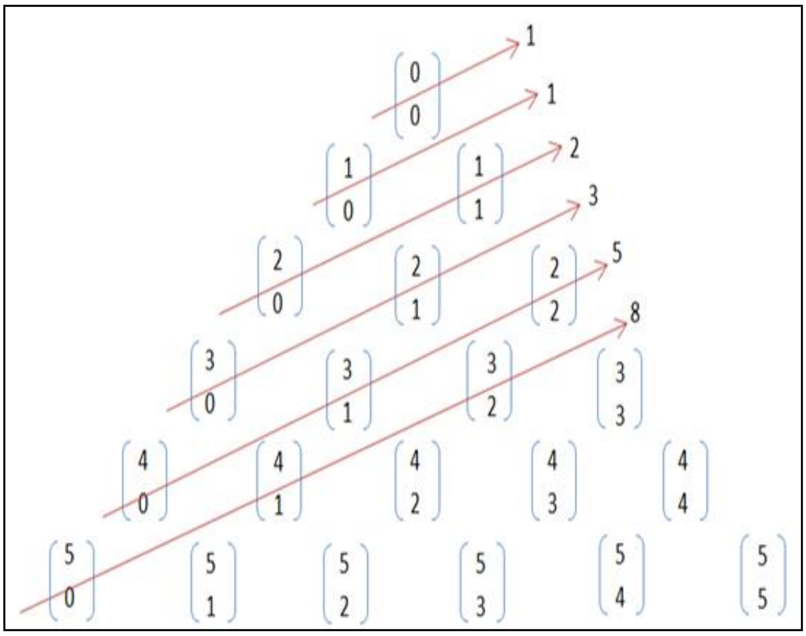

Figure 1: Pascal's triangle.

This study aims to express this existing image using a formula and to find the nth element of the Fibonacci sequence with less time complexity (time complexity) and computation time (computation time). Therefore, (n-1) is required to find the nth element of the Fibonacci sequence and there is no need to perform any computation after starting from element (n-2) up to the first element.
In order to find the coefficients in the expansion of (x±y)n, which is a binomial term, the coefficients generated using Pascal's Triangle are required. The conceptual expressions and techniques used to obtain and work with Pascal's triangle are briefly explained below.

THE GOLDEN RATIO AND THE GREAT PYRAMID

Fibonacci numbers have a different property. When a number in the series is divided by the number before it, very close numbers are obtained. After the thirteenth number in the series, this ratio always starts with 1.618 and reaches the golden ratio at infinity.
The method by which the Golden Ratio can be found on the radius of a circle is shown in the graph above. After accepting the diagonal (AC) of the rectangle TC AO as one of the sides of an isosceles triangle, triangle ABC is formed. When the height of the triangle is taken as 1, the OB side of the COB triangle becomes 0.618034, which is the Golden Ratio. In this context, the triangle is bisected by a line equal to the radius of the triangle, drawn perpendicular from T, the midpoint of the CGO side F, to A, the midpoint of the GO side (Figure 2);

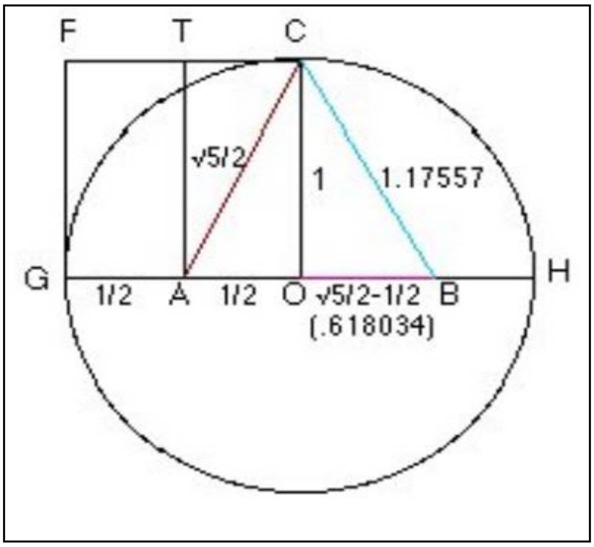

Figure 2: Triangular application of Fibonacci

Using a trigonometric ruler, we see that angle OCB is 31"43 and angle OBC is 58"17. The Egyptian priests may have found this much more important, provided they preserved the picture in the diagram above (Figure 3).

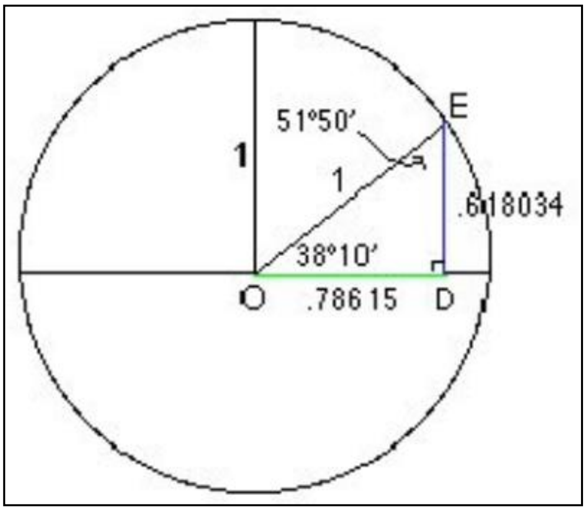

Figure 3: Application of Fibonacci in the Pythagorean triangle

In the diagram above, one of the sides of the triangle common to the right angle is 0.618034 of the radius, but this time the neighboring side is the hypotenuse. Here, using a trigonometric table, it can be discovered that the opposite angle of 0.618034 is 38"10 and the other angle is 51"50. The length of the OD side is also found to be 0.78615 of the radius using the Pythagorean Theorem. There are two important points that distinguish this construction from the others. First, the length of the ED edge (0.618034) is equal to the length of the OD edge (0.78615).

A man put two rabbits in a walled space. If one pair of rabbits gave birth to a new pair within a month and each new pair took a month to develop, how many pairs of rabbits will have been born between the four walls in a hundred years? (Figure 4).

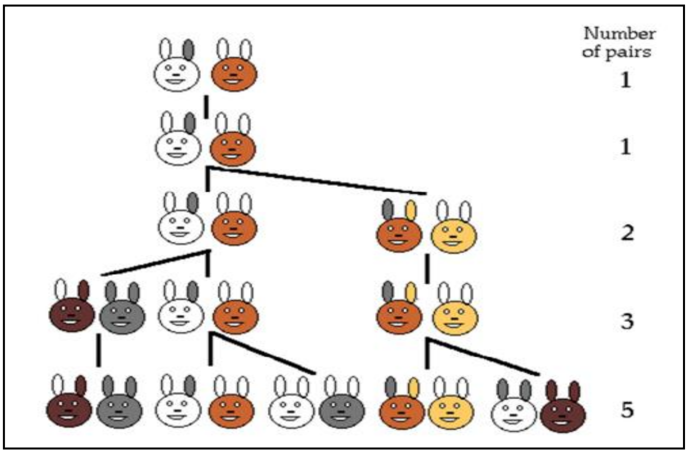

Figure 4: Fibonacci's sequence in rabbit offspring

Fibonacci included this issue in his book (which book), which he probably saw as a method of addition. However, a sequence of numbers appeared in this problem. The solution to the problem is as follows: At the end of the first month there is only one pair. . At the end of the second month, the female gave birth to a pair of cubs, which resulted in two rabbits. At the end of the third month, our first female gave birth to two litters, so we have three pairs of rabbits. At the end of the fourth month, our first female gives birth to two more litters, and the female born two months earlier gives birth to another pair. There are now five rabbits. At the end of one month, this equals the sum of the rabbit pairs of the two months before that month. Therefore, the answer is 354,224,848,179,261,915,075.

The results of this question are 1, 1, 2, 3, 3, 5, 5, 8, 8, 13, 21, 21, 34, 55, 89, ...

If the flower part of a sunflower or daisy is magnified, you will probably get an image close to the photo below. Counting the clockwise and counterclockwise spirals in the pattern in the figure yields the numbers 21 and 34. The ratio of these numbers is very close to the golden ratio (Figure 5).

Figure 5: Fibonacci sequence of sunflower.

This ratio is also seen in many works of Mimar Sinan. As a matter of fact, the golden ratio is also found in Turkish architecture and art. The Davut Pasha Mosque in Istanbul, the crown gate of the Ince Minareli madrasah from the Seljuks in Konya, and the Divriği Complex in Sivas, which survived from the Mengüçoğulları (Figure 6).

Figure 6: View of Selimiye Mosque

Divide a line segment at a certain point and let the ratio of the long segment to the short segment be equal to the ratio of its entire length. Thus,

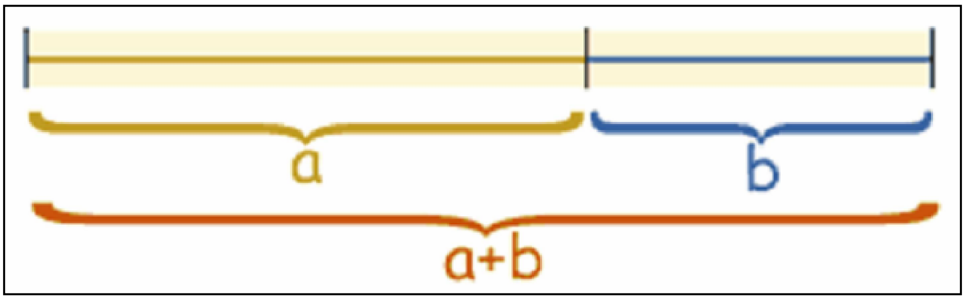

Figure 7: Fibonacci applied to a line segment.

For simplicity, let us prefer one of them, under these conditions, on the line segment indicated in the figure;

If (a+b) / a = a / b;
If a = x b = 1;
x2 = x + 1. This equation
When x2 -x - 1 = 0 is written as x2 -x - 1 = 0 and its roots are searched
x1= 1.6180339887 and x2= 0,6180339887
1,1,1,2,3,3,5,5,8,13,21,21,34,55,89,144.
initial values a(1)=1 and a(2)=1 and
The sequence defined by the reduction relation a(n)-a(n-1)-a(n-2)=0 is called Fibonacci sequence.
Also the special equation;
The roots of the equation, with λ^2-λ-1=0.
λ1,2=(1±√5)/2.
μn=A1((1+√5)/2)n + A2 ((1-√5)/2)n.

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By solving the equation $A1((1+\sqrt{5})/2)2 + A2 ((1-\sqrt{5})/2)2 = 1$

$$A1=1/\sqrt{5} ,A2= -1/\sqrt{5})$$

$$\mu n= 1/\sqrt{5}[((1+\sqrt{5})/2)2 + 1/\sqrt{5} ((1-\sqrt{5})/2)2] = 1$$

Of these roots, only the second root is not included in the solution set since no length is negative. The first root, however, gives the golden ratio, which we call phi (Fi). 1,6180339887... Everyone has an approximate golden ratio (Figure 8). For example, the area between the elbow and the wrist is 1.618. The first 20 terms of Fibanocci are determined as follows;

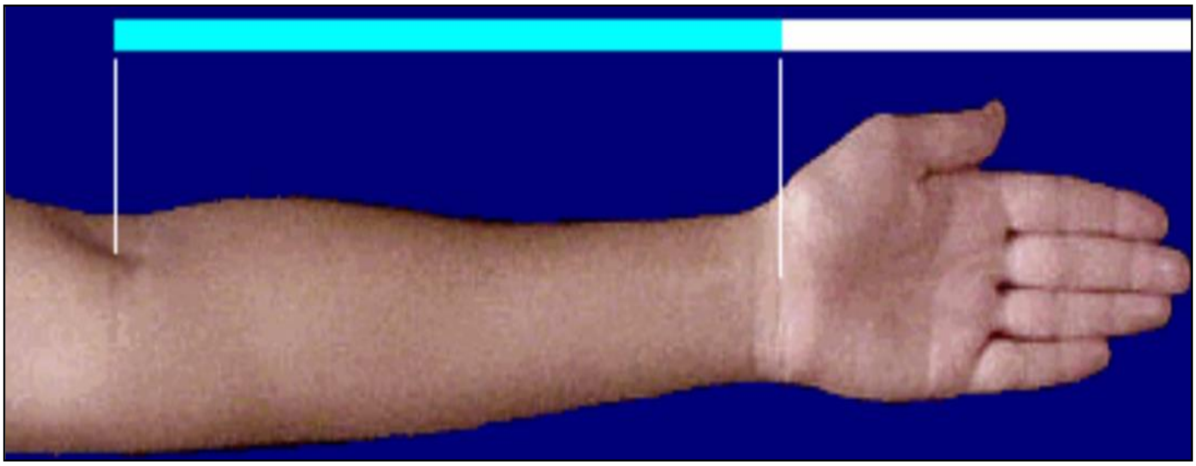

Figure 8: Illustration of the Fibonacci golden ratio (Whole bar / Turquoise part)

The Fibonacci sequence is particularly useful in the serialization of durations. Such as when the ratio between adjacent small durations is much more significant (Figure 9);

Figure 9: Fibanocci sequence in musical notation

It avoids the problem of total duration, (♪ : ♪ = 2:1), between adjacent large durations (♩. : ♩♪. = 12:11). The Fibonacci series also avoids other extremes of an equally tempered (i.e. geometric) series: For example, the interval between the smallest and largest durations is too large to be handled effectively, even in a series of only six durations. As in "Il Canto Sospeso"). Fibonacci numbers offer a sequence relationship as simple (and therefore perceptible) as the geometric and arithmetic sequences mentioned above, but the Fibonacci sequence plays an important mediating role between the extremes.
The Fibonacci line that would copy the first line form is omitted. Another observation is that the system (taken together with the serialization of the dynamics [see Stockhausen analysis *34]) generates the music completely automatically.

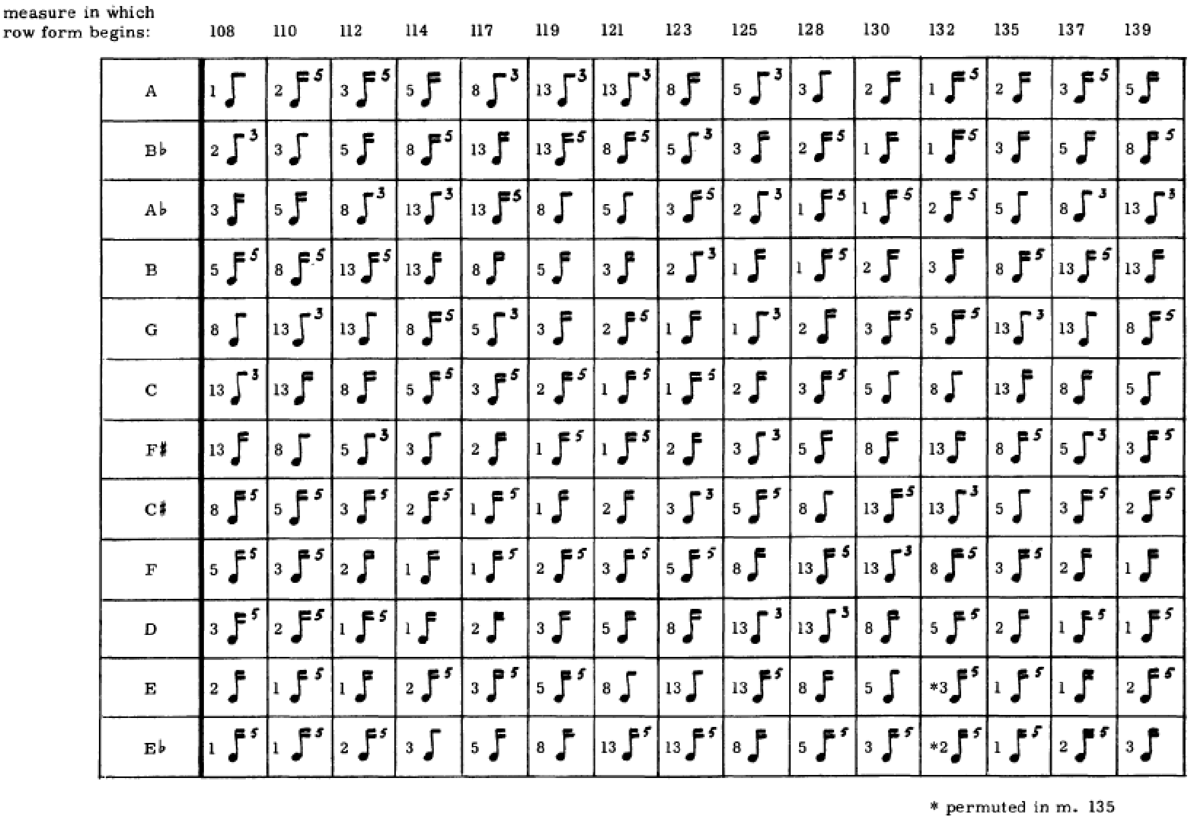

Figure 10: Fibonacci sequence in musical notes

When two or more voices end a duration simultaneously, the subsequent simultaneous attack allows the composer to freely choose which of the simultaneity notes to apply the available Fibonacci multipliers to (e.g. see measure 133 - C, C# or F~ extended 13, 13 or 8 times or any combination of I or I).

In conclusion, based on the formulas and figures above, the classical Fibonacci sequence is discussed in detail.

Another question that will make a significant contribution to the related literature and fill the research gap in the literature is as follows: "What will be the result if it is assumed that there are elements before the first elements of the Fibonacci sequence 0,1,1?" In other words, what will be the result if the sequence unfolds to the left instead of the right?

Let the numbers a,b,c be considered as the elements immediately preceding the elements 0,1,1. In this context, part of the array is a,b,c,0,1,1. According to our formula (each number is obtained by adding the preceding number to the next number) c + 0 = 1, so c = 1. As the pattern continues to the left by substituting the value of c, b=-1 since b+1=0 in the order a,b,1,1,0,1,1. According to the order a,-1,1,1,0,1,1, a + (-1) = 1, so a = 2.

If this is continued indefinitely, new Fibonacci numbers ... {-377, 233, -144, 89, -55, 34, -21, 13, -8, 5, -3, 2, -1, 1, 0, 1, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377 ...} will be obtained. This completes the fascinating part of the sequence. If this sequence is now divided into two parts, the Classical (F) and the New (G), the new part will no longer open to the left but to the right as a sequence.

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The elements of the classical part are F={0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 55, 89, 144, 233, 377 ...}
Elements of the new part G= {0, 1, -1, 2, -3, 5, -8, 13, -21, 34, -55, 89, -144, 233, -377 ...}
Since the elements of the new part are expanded to the right, the definition (summing each number with the number before it to get the next number) has become invalid. This definition has been updated to "the sequential difference of each number with the number before it to get the next number". In other words, our new pattern is;
0-1= -1
1-(-1)=2
-1-2=-3
2-(-3)=5 ... works as follows.
So it is time to write and discover the general term of the new Fibonacci sequence.

$$G_n = G(n) = \begin{cases} 0 & n=0; \\ 1 & n=1; \\ G(n-2)-G(n-1) & n>1. \end{cases}$$

So the sequence defined by the reduction relation a(n)+a(n-1)-a(n-2)=0 can be called the New Fibonacci sequence.

CONCLUSION
It is striking that the elements of the new Fibonacci sequence are parallel to the elements of the classical Fibonacci sequence and that the elements go positive-negative sequentially. In addition, the sequence Fn+Gn contains remarkable realities in the sequential sum of the elements. In this context, by utilizing the above functions;
The sequence Fn+Gn={2, 4, 10, 26, 68, 178, 466 ...} is obtained by summing the consecutive elements without writing the "0" elements. This is again a recursive sequence because each element is formed by taking three times the previous element and subtracting the two previous elements. At this point,
(3×4)-2=10,
(3×10)-4=26,
(3×26)-10=68,
(3×68)-26=178 ...
That is, the sequence Fn+Gn can be defined by the reduction relation a(n)=3×a(n-1)-a(n-2).
In addition, the sequence Fn-Gn in the sequential summation of the elements also contains very important facts. In this context, using the above functions;
The sequence Fn-Gn={2, 6, 16, 42, 110, 178, 288, 754 ...} will be obtained without writing the "0" elements that occur when subtracting the ordered elements from each other. We can easily see that we can find the elements here not by using the reduction relation but by using the elements of the array Fn+Gn. When we subtract the ordered elements in the Fn+Gn array from each other sequentially, we find the elements of the Fn-Gn array. Let's see
4-2=2
10-4=6
26-10=16 ...
68-26=42
More inferences can be easily obtained by doing the necessary studies. As a result, as can be seen, the Fibonacci sequence is more than thought.

If we return to the new Fibonacci sequence, when we look at the ratio of one element to the previous element, we see that it approaches the number -1.618, that is, the golden ratio value approaches the negative value. It is known that the classical Fibonacci sequence is effective in various fields such as Finance, Nature, Art, Cryptography, Music, Economics.

REFERENCES

[1] C.H. Hsu, and D. Hung-Son, "The application of Fibonacci sequence and Taguchi method for investigating the design parameters on spiral micro-channel." Applied System Innovation (ICASI), International Conference on IEEE, 2016.
[2] K. Plofker., and J. Hannah, "Mathematics in India," *Aestimatio: Critical Reviews in the History of Science*, vol. 7, pp. 45-53, 2015.
[3] N.S. Goel, and N. Richter-Dyn, “Stochastic models in biology,” Elsevier, USA, 2016.
[4] T.V. Brasch, J. Byström, and L.P. Lystad, "Optimal Control and the Fibonacci Sequence," *Journal of Optimization Theory and Applications,* vol. 154, no. 3, pp. 857–78, 2012.
[5] C. Orozco-Henao, "Active distribution network fault location methodology: A minimum fault reactance and Fibonacci search approach," *International Journal of Electrical Power & Energy Systems*, vol. 84, pp. 232-241, 2017.
[6] H. Kaplan, R.E. Tarjan, and U. Zwick, "Fibonacci heaps revisited," *arXiv preprint arXiv*, pp. 1407.5750, pp. 1-13, 2014.
[7] S. Klavžar, "Structure of Fibonacci cubes: a survey." *Journal of Combinatorial Optimizatio*, vol. 25, no. 4, pp. 505-522, 2013.
[8] A. Stakhov, V. Massingue and A. Sluchenkova, "Introduction into Fibonacci coding and cryptography," *Osnova, Kharkov*, vol.5, no.3, Feb., 1999.
[9] T. Lengyel, "A counting based proof of the generalized Zeckendorf's theorem," *Fibonacci Quarterly*, vol. 44, no. 4, pp. 1-2, 2006.
[10] D.E. Knuth, *The Art of Computer Programming*, Boston: Addison–Wesley, 1997.
[11] R.S. DeBellis, M.S. Ronald, and Phil C.Y., "Pseudorandom number generator." U.S. Patent No. 6,044,388. 28 Mar. 2000.
[12] M. Cohn, *Agile estimating and planning*. London: Pearson Education, 2005.
[13] T. Niemann, *Sorting and Searching Algorithms,* Oregon, 2010.
[14] M. Edson, O. Yayenie, "A New Generalization of Fibonacci Sequence & Extended Binet's Formula," *Integers,* vol: 9, no: 6, pp. 639-654, 2009.
[15] H. Berkmen, https://dusunebilen.wordpress.com/2013/07/24/altin-oran/
[16] https://tr.wikipedia.org/wiki/Leonardo_Fibonacci
[17] https://tr.wikipedia.org/wiki/Alt%C4%B1n_oran
[18] http://bugunkukonumuz.blogspot.com.tr/2012/10/dogann-dilinden-fibonacci-dizi-vealtn.html.